New EEM/BEM Hybrid Method for Electric Field Modelling in Cable Accessories

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Abstract — From the aspect of design and operation, cables and cable accessories are still very actual, although it has been more than a hundred years since the appearance of the first cable. Many questions concerning the construction of cables, cable accessories especially, are still open. In MV and HV systems, cable accessories are one of the weakest links. Cable terminations and joints are designed to eliminate the stress concentration at the termination screen. This paper presents one numerical method, based on combination of Equivalent Electrodes Method and New Hybrid Boundary Elements Method, which can be successfully applied on cable accessories design. Some different constructions were experimentally verified.

Keywords — Cable terminations, cable joints, deflector’s cones, equipotential surfaces, electric field distribution, equivalent electrodes method, hybrid boundary elements method.

I. INTRODUCTION

The medium and high voltage cable networks have an important role in electric energy distribution. From the aspect of failures, the most important parts of cable lines are the cable joints and terminations. The majority of cable failures on distribution system are caused by defects in the cable accessories. For that reason any improvement in their construction is of interest.

Electric field control and rigorous technological process are important for cable accessories reliability. Hot spots very often coincide with maximum electric field. In order to optimize the cable joints and terminations parameters, two criteria were monitored – total electric field magnitude, \( E_{\text{max}} \), and magnitude of the tangential component, \( E_t \).

Without control, the high stress can lead to partial discharge in the dielectric, ionization and breakthrough in the air or dielectrics, and rapid aging of the insulation, leading to a dielectric puncture and failure. Appropriate choice of dielectrics and shape of deflectors is the most important. Numerical calculations are base of that.

In combination with Equivalent Electrodes Method (EEM), Authors are proposed one new numerical method ([1], [2]), so called Hybrid Boundary Elements Method (HBEM). The basic idea of the theory is that an arbitrary shaped boundary between two dielectrics can be replaced by equivalent charges (ECH), where ECH are located at the dielectrics boundary [3]. It is possible, by using condition for the normal component of polarization vector, to form a system of linear equations, where equivalent polarized charges are unknown. By solving this system, the unknown charges can be determined [4]. Similar procedure can be applied on arbitrary shaped perfect conducting electrodes, where electrodes are replaced by finite system of Equivalent Electrodes (EEs) [5]. In contrast to Charge Simulation Method, where the fictitious sources are placed inside the electrodes volume, the EEs are located on the body surface. The radius of EEs is equal to equivalent radius of electrode part, which is substituted. In recent decades, meshless methods have attracted a growing attention from mathematics and engineering communities. Generally speaking, these methods can be divided in to the domain-type or boundary type techniques, depending on their basis functions and equations of interest.

The boundary element method (BEM) [6] is a numerical method for the solution of boundary integral equations, based on a discretization procedure. It is an alternative to the domain methods of analysis in electromagnetics, such as the finite difference method (FDM) or the FEM. The basis of the method is to transform the original partial differential equation, or system of PDEs, that define a given physical problem into an equivalent integral equation (or system) using Green’s function.

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Combined with the EEM [5], the hybrid BEM can now solve large-scale problems in electromagnetics. This opened up a wide range of applications for the HBEM. Some cable terminations and joints constructions are presented to demonstrate the efficiency, accuracy and potentials of the HBEM.

Stress distribution control is usually based on geometrical regulation [7]-[13] with the stress relief cones, special materials of high relative dielectric constant [14]-[19], or embedded electrodes system application [20]-[23]. There is no universal termination or joint. There is a variety of different types of termination and joints each with advantages and disadvantages. The optimization of cable terminations and joints are achieved by considering various constructions [24], [25].

The use of geometrically modelled cable accessories for terminating and jointing various shielded power cable types has been well documented in the literature [1], [2].
Cable breakdown most often happens because of a strong electric field in the cable insulation, close to the cable screen end. Commonly, it is controlled by deflector’s cones, conventional stress relief cones, which are geometric solution to the problem. Recent papers initiated a study of a new cable termination construction [15], [16]. Although the stress relief cones offer good solution for the electric stress reduction, the high relative dielectric constant extruded tubes, have many advantages in fabrication and installation. The tubes are easily made and can be fabricated in shape of strips or tapes with thick layers. Numerical program allowed the study of optimal relative dielectric constant and thickness. Cable termination should be made of material with relative dielectric constant $\varepsilon_r = 10$ or higher. The electric field at the cable termination was controlled by high permittivity material, but the results were not completely satisfied and further investigations were needed.

II. ELECTRIC FIELD DETERMINATION AT NON-MODELLED CABLE TERMINATIONS

Due to limitations in the production, delivery and setting up power cables, they are produced and delivered in several separate lengths. Cable itself, consists of two cable terminations and arbitrary number of cable joints, depending on the cable route length, which may be several kilometers long. The cables are typically produced up to 1 km long sections, wound on the cable drum. The cable terminations are assembled at the ends of cable line and can be either for indoor or outdoor mounting. There are many techniques in the practice for jointing and terminating the power cables, but the most preferable is heat shrinkable (HS) one.

Design of electrical equipment was originally based on analytically derived formulas. However, as the need arises for materials with better physical limitations and for design optimizations, more sophisticated design tools are required. Numerical methods developed as the basis of design tools since they are capable of modelling accurately complex geometry, non-homogenous regions, different types of excitation and non-sinusoidal quantities that analytical techniques are incapable of. Finite Elements Method ([4], [13], [16]-[20], [24], [25]) has become the most popular method for many years for the analyses of electromagnetic problems in all types of electrical apparatus.

Due to the symmetry of the problem, by adopting a cylindrical reference system with the z-axis coincident with the axis of the conductor, an equivalent two-dimensional problem can be studied. Region between interior conductor (having radius $a$), 1, and the exterior conductor (having radius $b$) is filled with dielectric ($\varepsilon_r$). Axial cross-section of equipotential surfaces for cable terminations without cable insulator, and with one is presented in Fig. 1 and Fig. 2.

Electric potential and electric field at air cable accessories, for this cable geometry, are determined by used only EEM.

When the power cable is cut off, the conductor of the cable is exposed to the air, the voltage potential of which is 100% high voltage. The metal shielding enclosure is also exposed to the outside, the voltage potential of which is 0. The rubber-plastic electric shield and the insulating layer between the conductor and the metal shielding enclosure are stripped off by a predetermined distance. 35 kV cable (XHE 49-A, 1x150/25 mm², 20/35), produced in Holding company “Fabrika kablova Jagodina”, is analyzed. Radius of inner conductor is $a=762 \text{mm}$, outer is $b=17.5 \text{mm}$, and $E_0 = \frac{U}{a} = 4.58 \text{MV/m}$. The electric field distribution in cable terminations is strongly non-uniform. 3D electric field distribution at the end of this cable is presented in Fig. 3.

III. ELECTRIC FIELD DETERMINATION AT MODELLED CABLE TERMINATIONS

Cable termination should be made of material with relative dielectric constant $\varepsilon_r = 10$ or higher. The thickness of the layer $d = 1 \text{mm}$ was found to be sufficient
and there was no point in increasing it. The same may be achieved with the layer of relative dielectric constant \( \varepsilon_r = 5 \), but the thickness must be greater. The increased relative dielectric constant layer of 10 cm as the cable termination was found to be sufficiently long.

Charge density per unit surface is constant in the distant regions from cable breaks. Appropriate electrical field is:

\[
E_{\text{hom}} = \frac{aE_0}{r \log \frac{b}{a}}
\]

where \( E_0 = \frac{U}{a} \), and \( U \) is voltage the coaxial cable is supplied with.

If it is presumed that such charge distribution is also in the surroundings of the cable break (Fig. 1a), and:

\[
g(C, L_n) = L_n + \sqrt{C^2 + L_n^2} ;
\]

\[
A^2 = r^2 + a^2 - 2a \cos \theta' ;
\]

\[
B^2 = r^2 + b^2 - 2b \cos \theta' ;
\]

\[
I(r) = \begin{cases} 
\ln \frac{b}{a}, & 0 \leq r \leq a \\
\ln \frac{a}{b}, & a \leq r \leq b \\
0, & r \geq b 
\end{cases}
\]

where \( r, \theta \) and \( z \) are cylindrical coordinates, the approximate expression for potential, in different regions, is:

\[
\frac{\varphi_{\text{ap}}(r, z)}{U} = \frac{1}{2\pi} \int_0^\pi \ln \left( \frac{g(B, L, z - L)}{g(A, L - z)} \right) d\theta'
\]

for \( z \leq L_1 \leq L_2 \),

\[
\frac{\varphi_{\text{ap}}(r, z)}{U} = \frac{1}{2\pi} \int_0^\pi \ln \left( \frac{g(B, L, z - L)}{g(A, L - z)} \right) d\theta'
\]

for \( L_1 \leq z \leq L_2 \),

\[
\frac{\varphi_{\text{ap}}(r, z)}{U} = \frac{1}{2\pi} \int_0^\pi \ln \left( \frac{g(A, L - z)}{g(B, z - L)} \right) d\theta'
\]

for \( L_2 \leq z \leq L_1 \), and:

\[
\frac{\varphi_{\text{ap}}(r, z)}{U} = \frac{1}{2\pi} \int_0^\pi \ln \left( \frac{g(A, L - z)}{g(B, z - L)} \right) d\theta' + 2\pi I(r)
\]

for \( L_1 \leq L_2 \leq z \).

On the basis of the expressions for potential (5-8), and for \( L_1 = L_2 = L \), approximate expressions for electric field’s radial and axial components are determined.

Real expressions are superposition of approximate expressions and additional terms that originate from equivalent electrodes (at boundary electrode-air (dielectric)) or equivalent charges (at boundary dielectric-dielectric).

It is possible, using condition that the normal component of polarization vector is different on boundary of two dielectrics [3],

\[
P_{1n} - P_{2n} = \eta_P ,
\]

to form a system of linear equations, where polarized charges are unknown. By solving this system, the unknown charges can be determined. Polarized charges, situated on the boundary between two dielectrics, now present equivalent electrodes, placed in free space.

### A. Refractive modelled cable terminations

An efficient and flexible model, based on the electric quasi stationary formulation of the problem, has been proposed to evaluate the electric field distribution in refractive modelled cable terminations. Two-dimensional domains, even of very complex shapes and the finite thickness of the grading materials are properly taken into account, allowing an accurate evaluation of the electric field distribution.

![Fig. 4. Refractive modelled cable termination.](image)

On the basis of the expressions for potential (5-8), and for \( L_1 = L_2 = L \), approximate expressions for electric field distribution in refractive modelled terminations are obtained.

Axial cross-section of equipotential surfaces for refractive modelled cable termination (Fig. 4) is shown in Fig. 6.

![Fig. 6. Axial cross-section of equipotential surfaces for refractive modelled CT.](image)

B. Geometric method for cable terminations modelling

Geometric stress control involves an extension of the shielding (Fig. 7) which expands the diameter at which the terminating discontinuity occurs and thereby reduces the stress at the discontinuity. It also reduces stresses by enlarging the radius of the shield end at the discontinuity. Using the HBEM [1]-[2] and EEM [3] it is possible to determine potential and electric field in arbitrary chosen point of cable end region.

Equivalent sources (Fig. 8), which replace various segments of cable conductor ends and boundary between dielectrics, have toroidal shape.
C. Combined modeling of cable accessories

In spite of trends that main parts of the cables accessories shall be prefabricated, to avoid possible influence of human factor on preparing the cable ends for jointing and terminating, there still must be manual work during the installation process. Taking into account that the jointer work cannot be perfect during the assembling process, some microscopic air bubbles necessarily could remain in the interface between dielectric layers, causing the local discharges under the both electric and thermal field.

One new design of cable termination, to avoid partial discharges at the boundary deflector-dielectric, is presented in Fig. 11. Region between interior conductor and the exterior conductor is filled with dielectric ($\varepsilon_r$). Deflector is placed in refracting dielectric ($\varepsilon_{pp}$) with very high values of permittivity ($\varepsilon_{pp}$).

Electric field distribution on outer conductor for 35 kV cable, where are: $b = 3a$, $\delta = a/10$, $r_1 = 4.3a$, $z_{11} = 0$, $z_{d1} = 4a$, $z_{d2} = 8a$, $z_{12} = 12a$, $z_{c} = 32a$, $z_{13} = 37a$, is shown in Fig. 12. Curve 1 presents numerical results, and set of points 2 presents measured values.

### Table 1: Compared numerical results of maximal electric field intensity with FEM.

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<th>$N_1$</th>
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<th>$N_4$</th>
<th>$N_{d1}$</th>
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**FEM**

2.26009761
Values for electric field intensity obtained by using the FEM and our new HBEM are compared and results are shown in Table 1.

At the same time, Alex Pokryvailo et al published paper [16], where they proposed completely same construction of cable termination. Experimental results confirm our numerical values.

Similar considerations can be applied on cable joints (Fig. 13). The relative permittivities of insulation rubber and polyethylene were assumed to be 2.5 and 2.3, respectively.

![Fig. 13. Refractive+geometrically modelled cable joint.](image)

Axial cross-section of equipotential surfaces for refractive and geometrically modelled cable joint, where deflector’s cones end is modelled in accordance with either exponential, polynomial or ellipsoidal function [7]-[12] is shown Fig.14.

![Fig. 14. Axial cross-section of equipotential surfaces for refractive+geometrically modelled cable joint.](image)

The excellent electrical properties of cross-linked polyethylene, in combination with its good physical properties, have attracted many manufacturers worldwide to consider its application for high voltage direct current (HVDC) underground power cables. The operation of polarity reversal in DC transmission system may create extremely high electric stresses in certain parts of the insulation when space charge is present in cable insulation.

Charge distribution at outer conductor (1), dielectrics boundary (2) and deflector (3) is presented in Fig. 15.

![Fig. 15. Charge distribution at outer conductor (1), dielectrics boundary (2) and deflector (3).](image)

Density of equipotential lines is much lower and intensity of critical electric field is less then one compared to non modelled cable.

Same deflector’s shape can be applied on cable joints (Fig. 17).

![Fig. 17. Refractive+geometrically modelled cable joint.](image)

Equipotential lines for this modelled cable joint are shown in Fig. 18.

![Fig. 18. Equipotential lines.](image)

The spacing of the electric flux lines and the corresponding equipotential lines is closer in the vicinity of the conductor than at the shield, indicating a higher electric stress on the insulation at the conductor. This stress increase, or concentration, is a direct result of the geometry of the conductor and shield in the cable section and is accommodated in practical cables by insulation thickness sufficient to keep the stress within acceptable values.
Electric field distribution (3D) at the cable joint, using deflector with exponential shape, is presented in Fig. 19.

**Fig. 19. Axial component of electrical field distribution.**

**D. Application of multilayer dielectric systems on electric field modelling in cable accessories**

Electric field distribution is considered for multilayer dielectrics [26]-[27] with constant dielectric permittivity. A comparison of the electric scalar potential distribution when multilayer dielectric system is applied, and without this system, is presented.

Capacitive stress control consists of a material possessing a high dielectric constant, generally in the range of 30 and also a high dielectric strength. This is generally an order of magnitude higher than the cable insulation. Located at the end of the shield cut-back, the material changes the voltage distribution in the electrical field surrounding the shield terminus.

Cable termination, modelled by four layer dielectric system and very thin deflector’s cones (conventional stress relief cones) is shown in Fig. 20.

**Fig. 20. Modelled cable termination by four layer dielectric system and very thin deflector’s cones, having ellipsoidal form.**

Multilayer dielectric systems can be applied on electric field reduction at cable joints, too. One possible construction of cable joint with reduced values of electric field is shown in Fig. 22.

**Fig. 22. Modelled cable joint by using five - layer dielectric system and very thin deflector’s cones, having polynomial form.**

Electric scalar potential and intensity of electric field at jointing region are shown in Fig. 23 and Fig. 24.

**Fig. 23. Equipotential curves at modelled cable joint by using five - layer dielectric system and very thin deflector’s cones.**

**Fig. 24. Electric field distribution, \( E \, [MV/m] \), in radial direction, \( r/a \), for \( r/a = 3.0 \) (curve 1), \( r/a = 4.0 \) (curve 2), \( r/a = 5.0 \) (curve 3).**

**E. “Snail” terminations**

The effects of electro-thermal [16] and mechanical stresses can be enhanced in the presence of interfaces that may, thus, become the weakest points of the insulation system, both in AC and DC. Special attention must be paid to the interface between cable and joint body. The electrical field along this interface (part of the field parallel to the interface) is always a critical issue as the dielectric strength of this interface is practically lower than the strength of an insulating body. Therefore the stress control elements must be designed that way, which the field along this interface stays within the permissible limits. Interfaces can act as a trigger for partial discharges.
(PD) when the contact between surfaces is not well made, and such activity should be strictly avoided for cable and accessories.

One possibility for cable termination design with discretized stress grading dielectrics is shown in Fig. 25.

![Fig. 25. Modelled cable termination, having snail form.](image)

Equipotential lines are shown in Fig. 26, and radial component of electric field distribution, $E_r$, in radial direction, $r/a$, for $z/a = -1$ (curve 1), $z/a = 1$ (curve 2), $z/a = 2$ (curve 3), is presented in Fig. 28.

While the resistive and refractive method is successfully used for medium voltage applications up to 72.5kV maximum, the geometrical field control method is the standard method for high voltage and extra high voltage applications. Electric field modelling by a well defined contour still offers the best quality from design and production point of view.

During design stage, new HBEM is an important tool and offer a vast range of possibilities, like:
- Calculation of the electrical field in any direction of the joint body
- Optimization tools for calculating the optimum shape of stress control elements
- Solving of coupled fields, like thermo-mechanical stresses
- Models for non-linear behavior of materials, like stresses in polymeric materials
- Simulation of slip-on procedures

IV. CONCLUSION

To ensure reliable performance over the lifetime of the cable system, basic design concepts must be correct. Two typical types of the cable termination mechanisms are explored by using the simulation program. The first type is the deflector model. It disperses the concentrating electric field at the termination area with its geometrical structure. The results of the analyses show that the electric field is decreased with the curved geometry of the deflector. The other type of the model is the stress control tube consisting of a layer with high relative permittivity. The stress control layer applied on the cable screen reduces electric field at the termination area successfully.

The design and choice of materials [25]-[28] of a cable accessory are vital to ensure its adequate performance during a long service life. Evaluation of material properties must always address the compatibility with the other materials that it will be in contact with. The ultimate performance and reliability of a termination depends on its materials, design, installation and operating environment. New materials [29]-[30] and design were required to achieve the target improvements. The paper describes design and performance results for new modelled terminations. It was presented a method, which allows computing the electric potential and field in 2D insulating structures, in presence of thin layers. A comparison of the electric field distribution in the case with control tube and without control tube is presented. Such a design could reduce the manufacturing cost, lower the difficulty of manufacturing, and reduce the time for installation.

The optimized stress control tube application technique possibly with non linear characteristic material or multiple stress control layers are suggested as the future work for the most satisfactory and economic way of the stress
controlled cable termination model. A new product has to undergo different tests before it can be used in real applications. After internal design tests are passed successfully an important part to check the integrity of the design and to get the acceptance by customers are type tests as specified in IEC60840 (60kV up to 170kV), IEC62067 (for voltages ≥ 170kV) or alternatively in IEEE404. However all the type tests specified above give only information about the design and the quality of a limited amount of samples. When a type test is passed it does not automatically mean that all the products produced afterwards in series are of sufficient quality.

High temperature superconductivity (HTS) materials can be applied on cable accessories design. HTS wires offer higher performance with reduced electrical losses, compared to conventional copper and aluminum wires.

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