Diakoptic Approach with Nested Partitioning for Electromagnetic Analysis

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Abstract — **We present nested partitioning of a given electromagnetic system when applying the diakoptic analysis. An electrostatic example is given, which illustrates the principle.**

Keywords — **Diakoptics, Nested partitioning, Method of moments.**

I. Introduction

THE diakoptic approach has recently been introduced into the analysis of large and complex electromagnetic (EM) systems [1]. This approach is based on partitioning the analyzed system into several subsystems, which are analyzed independently to obtain a matrix representation of each subsystem. These representations are then combined to obtain the final solution for the EM field in the whole system.

The diakoptic approach has been successfully applied to the analysis of various two-dimensional (2-D) and three-dimensional (3-D) systems, including problems of electrostatic, quasistationary, and dynamic fields [2]-[5]. In all cases considered, a dramatic reduction was demonstrated of the CPU time and memory requirements compared to the classical approach.

The diakoptic approach is convenient for the analysis of EM systems that consist of regions with many details, where the EM field has rapid spatial variations, separated by regions where the field is a slowly-varying function of coordinates. The partitioning of the system is best performed by placing subsystem boundaries in the regions with slowly-varying fields. A particularly convenient situation for applying the diakoptic approach is when the given system can be separated into congruent subsystems, because only a single analysis of such subsystems is required. However, the diakoptic approach can be applied to situations where the subsystems are different, as well.

Certain EM problems have a structure that is convenient for nested partitioning (hierarchical decomposition) [6]. Such a partitioning can further improve the speed of the numerical analysis. The main motivation for the presented work is to show that the diakoptic analysis can be efficiently applied to structures convenient for nested partitioning.

This work was supported in part by the Serbian Ministry of Science and Technological Development under grant TR 11021, by the COST action IC0603, and by the project F-133 of the Serbian Academy of Sciences and Arts.

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In this paper, we introduce a blend of the diakoptic approach with the nested partitioning. We outline the sequence of steps necessary for such analysis in the general case. We illustrate the technique on a 2-D electrostatic example.

II. Diakoptics

The diakoptic approach in the analysis of EM fields is based on the surface equivalence theorems [7], which enable dividing the space into two independent subproblems. The diakoptic approach consists in separating the given EM system into a number of nonoverlapping subsystems. The union of domains of all those subsystems is the domain of the original EM system.

Each subsystem has a closed boundary surface (termed as a diakoptic surface) which wraps it up (Fig. 1). The domain of a diakoptic subsystem is finite, and it is enclosed with the diakoptic surface. An exception is the outer subsystem, which exists in open-region problems (e.g., antenna analysis), which has an infinite domain. The inner subsystems may touch each other (as in Fig. 1), or may be separated.

According to the equivalence theorems, to enable such partitioning, so to keep intact the EM field within all subsystems, fictitious surface sources (equivalent sources) should be placed on the diakoptic boundaries. In electrostatics, these sources are surface charges and surface dipoles. In dynamic fields, the equivalent sources are surface electric and magnetic currents.

We consider here linear EM systems, filled with piecewise-homogeneous and isotropic media. In a numerical simulation, e.g., using the method of moments (MoM) [8], all EM field sources are approximated in a convenient manner. This approximation always includes a discretization: the field sources are represented in terms of a finite number of unknown coefficients.

A similar discretization is also applied to the sources at the diakoptic boundaries. Due to the linearity, the coefficients for the expansions of these surface sources for each subsystem are interrelated by a linear matrix relation, which is unique for the given subsystem. This matrix relation depends only on the EM properties of the subsystem, and does not depend on the environment in which the subsystem is located. Such a matrix relation is formally similar to the description of an electric network in terms of, for example, its admittance parameters.

Once such matrix representations for all subsystems are computed, they are combined in a global system of linear equations (which is referred to as the diakoptic system). The solution of this system yields the equivalent surface sources. Hence, the EM field at any point of the given EM system is determined, and various quantities of interest can be computed (e.g., impedance parameters, radiation pattern, etc.).

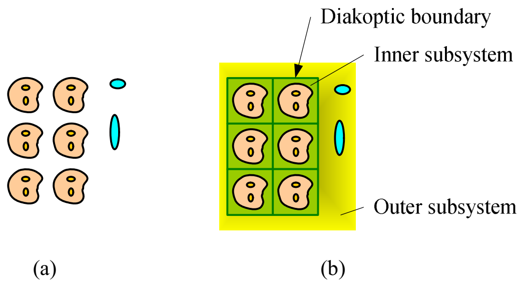


Fig. 1. (a) An electromagnetic system in free space and (b) its partitioning into subsystems.

III. NESTED PARTITIONING

The idea of nested partitioning can be explained on the example shown in Fig. 2a. The EM system under consideration can be partitioned into several subsystems at the first level, as shown in Fig. 2b. Thereby, some objects may remain in the outer subsystem (like the pink object on the left). It is desirable that the subsystems are congruent, so that the diakoptic approach is maximally efficient, though it is not an absolute necessity.

Each subsystem can further be partitioned into smaller subsystems, at the second level (Fig. 2b). Again, some object may remain outside the level 2 subsystems (like the turquoise objects on the right).

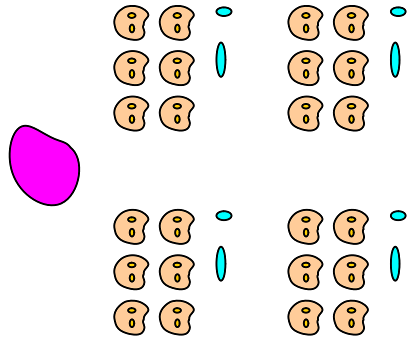
This nested partitioning can be carried out further, to inner-level subsystems, as appropriate by the geometry considered.

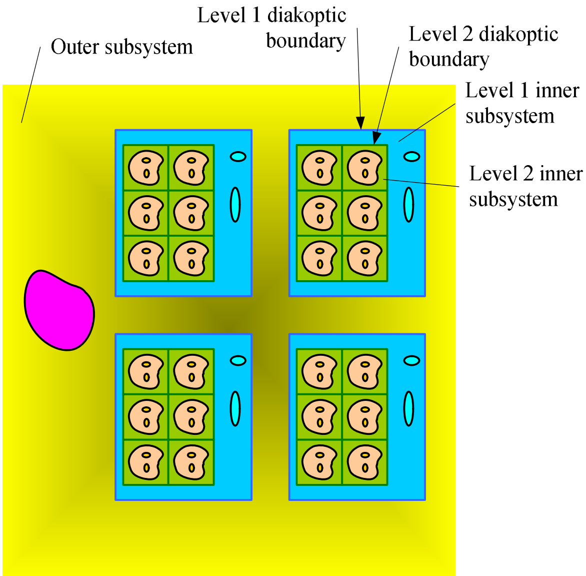
Following the diakoptic approach, each subsystem shown in Fig. 2b can be represented by an appropriate matrix, in the same way as a network in the circuit theory. An example of such a matrix representation is the Norton equivalent representation (i.e., the admittance parameters).

When all subsystems are assembled, a circuit is obtained as in Fig. 3. Each block in this figure is a network that represents one diakoptic subsystem. Level 0 represents the outer subsystem, and level 1 and level 2 networks correspond to the level 1 and level 2 inner subsystems in Fig. 2b.

Each block can contain independent excitations. Practical examples of such excitations are given potentials of conductors in electrostatics and electromotive forces of generators driving an antenna array. Alternatively, the independent excitations can be taken out of the corresponding network and attached to appropriate ports.

Similarly, the response of the system can be a quantity (electric or magnetic field, current, voltage, etc.) that is hidden within a network, or it can be represented by introducing an external port. Lines in Fig. 3 represent wires that interconnect the network ports. Currents and voltages at the ports correspond to the equivalent surface sources on the diakoptic surfaces in Fig. 2b.


(a)



(b)

Fig. 2. (a) An electromagnetic system in free space and (b) its nested partitioning.

The basic feature of the circuit shown in Fig. 3 is that the networks are interconnected in a hierarchical manner. A network in one level is connected only to networks that are one level lower (if such networks exist) and to one network that is one level higher (if such network exists).

The diakoptic system of equations corresponds to equations that are used to solve the circuit in Fig. 3. For example, these can be equations based on the nodal potentials. If this system is set for the whole circuit, a sparse system of linear equations is obtained. Alternatively, the circuit shown in Fig. 3 can be solved by topological elimination of unknowns. This approach consists in obtaining an equivalent representation (e.g., Norton representation) of a network looking into the ports on the right-hand side, level by level, starting from the level with the second highest index.

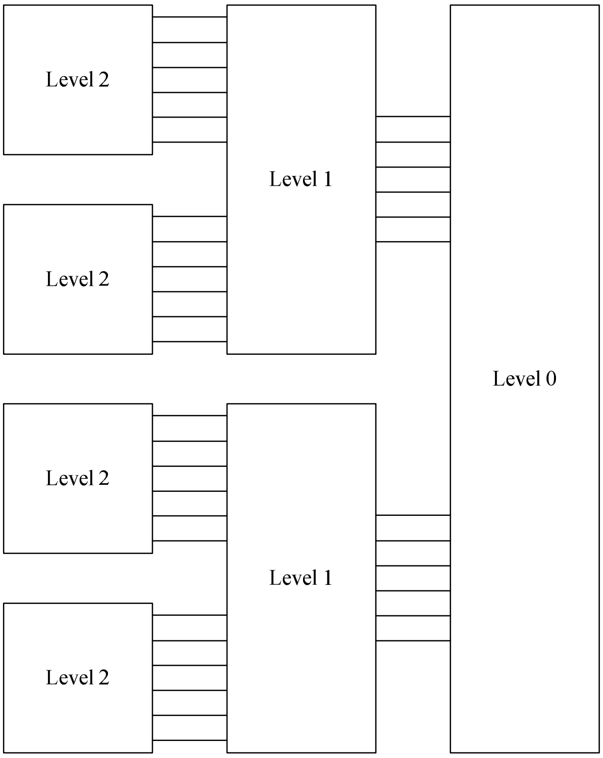


Fig. 3. Circuit-theory representation of the partitioned system shown in Fig. 2b.

IV. Example

The proposed technique is demonstrated on a 2-D electrostatic example. The basic building element of the analyzed system is an infinitely long conductor whose cross-section is shown in Fig. 4a. The circumference of this cross-section is given, in polar coordinates (r, ϕ), by

$$r(\phi) = 4 + \sin 8\phi + \cos 16\phi + \sin 24\phi + \cos 2\phi, \quad (1)$$

where r is in millimeters, and ϕ in radians.

The analyzed system consists of four such parallel conductors, located in a vacuum above a ground plane (Fig. 4b). The conductors are arranged in two pairs. One pair is obtained by rotating the other pair for $\pi/2$. Given are conductor potentials: the lower-left conductor is at a potential of 1 V with respect to the ground plane, and the potentials of all other conductors are zero. The objective is to evaluate the potential at the field point, shown in Fig. 4b.

In the classical MoM analysis, the circumference of each conductor is divided into 360 sections. Such a large number of sections is necessary to adequately model the geometry of the conductor. Each section corresponds to a flat, infinitely long strip. The charge distribution on each strip is approximated by a constant, and the point-matching testing is implemented as the simplest option. The total number of unknowns is 1440. The CPU time required to solve the problem on a notebook computer is 4.83 s. The result for the potential at the field point is 91.51 mV.

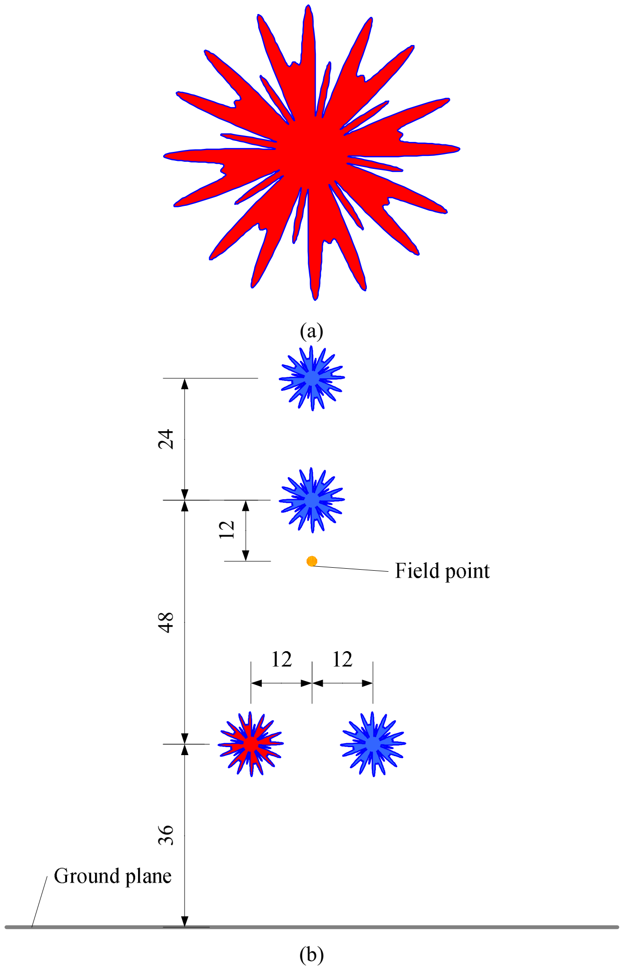


Fig. 4. (a) Cross section of a conductor and (b) arrangement of four conductors above ground plane with nested partitioning. Dimensions are in millimeters.

For the diakoptic analysis, the analyzed system is divided into nested subsystems, as shown in Fig. 5. Level 1 subsystems are congruent. Each subsystem encompasses a pair of level 2 subsystems. The corresponding diakoptic boundary is a circular cylinder, of radius 24 mm. Level 2 subsystems are also congruent. Each subsystem encompasses one conductor, and the diakoptic boundary is a circular cylinder of radius 8 mm. For both levels, each diakoptic surface is divided into only 36 sections.

The CPU time required for the diakoptic approach is 0.22 s, on the same computer, and using the same IMSL procedure for solution of linear equations. This is about 22 times faster than the classical solution. The resulting potential is 91.64 mV, which differs from the classical solution for only 0.14%.

In the classical solution, the largest storage is required for the system matrix (1440 unknowns). In the diakoptic solution, the largest system matrix is required for a level 2 subsystem (396 unknowns). Hence, the memory requirements for the diakoptic solution are smaller about 13 times than for the classical solution.

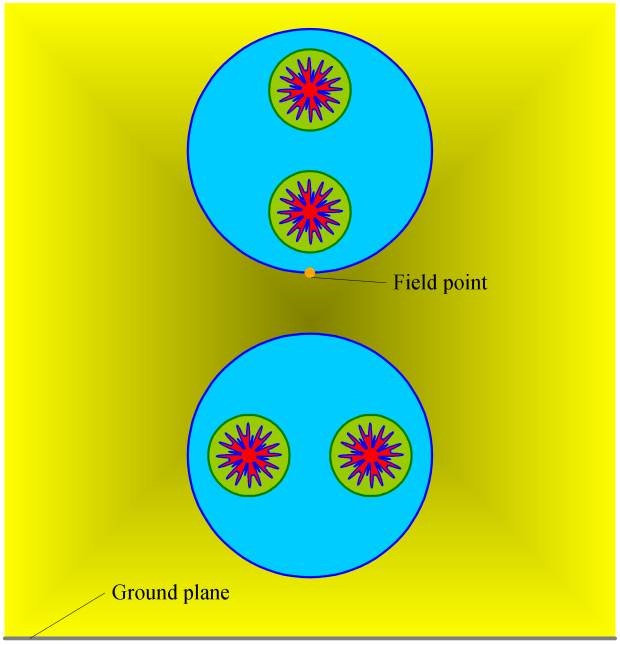


Fig. 5. Nested partitioning of the system shown in Fig. 4b.

V. Conclusion

We have proposed nested partitioning of large and complex electromagnetic systems, associated with the diakoptic approach. On a simple numerical example, we have demonstrated the tremendous reduction of the CPU time and storage requirements when compared to the classical numerical solution. With increasing the number of subsystems (e.g., having more conductors than in Fig. 2b), the advantages of the diakoptic approach further improve.

The nested partitioning is advantageous over simple partitioning when the simple partitioning results in a large number of subsystems. The nested partitioning essentially reduces the CPU time required to solve the diakoptic system of equations.

References

[1] A.R. Djordjević and D.I. Olćan, "Diakoptic analysis of large 2D electrostatic problems", Proc. of XIII Telfor, Belgrade, November 2005.

[2] D.I. Olćan, I.M. Stevanović, J.R. Mosig, and A.R. Djordjević "A diakoptic approach to analysis of large 2D problems", *2006 ACES Dig.*, pp. 527-531, Miami (FL), March 2006.

[3] D.I. Olćan, I.M. Stevanović, J.R. Mosig, and A.R. Djordjević, "Diakoptic approach to analysis of multiconductor transmission lines", *Microwave and Optical Technology Letters*, Vol. 50, No. 4, pp. 931-936, April 2008.

[4] D.I. Olćan, I.M. Stevanović, B.M. Kolundžija, J.R. Mosig, and A.R. Djordjević, "Diakoptic surface integral-equation formulation applied to large antenna arrays", *IEEE AP-S Int. Symp. 2008*, IEEE, San Diego (CA), July 2008.

[5] D.I. Olćan, M.M. Ilić, B.M. Notaroš, B.M. Kolundžija, and A.R. Djordjević, "Diakoptic Higher-Order FEM-MoM Approach, *IEEE AP-S/URSI Int. Symp. 2010*, July 11-17, Toronto, Ontario, Canada.

[6] C.C. Lu, W.C. Chew, "The Use of Huygens' Equivalence Principle for Solving 3-D Volume Integral Equation of Scattering", *IEEE Transactions on Antennas and Propagation*, Vol. 43, No. 5, pp. 500-507, May 1995.

[7] R.F. Harrington, *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961.

[8] R.F. Harrington, *Field Computation by Moment Methods*, IEEE Press, Piscataway (NJ), 1993.