Qualitative Analysis of Texture of Room Impulse Response using Fractal Dimension

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Abstract **— Texture is defined in acoustics as a subjective measure described with three qualities regarding reflectograms. Here we propose using fractal dimension for texture analysis, thanks to ability of fractals to reveal hidden phenomena within the signal. Further discussion towards quantitative descriptors of texture, based on obtained results, will be provided.**

Keywords **— Fractal dimension, reflectogram, room impulse response, fractal dimension, fractals.**

I. Introduction

In this paper we try to analyze texture as a subjective parameter of phenomena in room impulse responses (RIR). According to texture definition given in [1] it should be observed in the early part of reflectogram while high-quality texture requires a large number of early reflections, uniformly but not precisely spaced apart, allowing no one to dominate. Due to waveforms of RIRs (see Fig. 2. and Fig. 3. for instance) it is not an easy task to find reliable quantitative measure for texture.

In [2] texture parameter is defined as the number of the peaks higher than Schubert curve in the first 80 ms of RIR. This method uses Hilbert transformation for envelope extraction. Since reflectograms are signals with high variance changes, Hilbert transform usually does not give preferrable results without preprocessing.

However, visual inspection of logarithmic scale of RIR is common in practice. All criteria in abovementioned definition of texture can be condensed in a term of space-filling property of reflectograms. It transfers RIR analysis in the field of fractals.

Mandelbrot [3] defined fractals as sets (curves representing RIRs, in our case) whose Hausdorff dimension is not an integer, while Hausdorff dimension deals with tiling the set (i.e. planar curve here). Hausdorff measure is impractical to calculate and many other definitions of fractal dimension (FD) are defined; some of them will be presented in this paper. Generally speaking, FD reflects signal complexity. It will be later seen, through different definitions of FD, that complexity can be viewed as a space-filling property of the curve (i.e. signal).

We will not examine whether signal is fractal or not. Indeed, all fractals in nature manifest self-similar (fractal) behavior only in limited range of scales. Instead, we will restrict our analysis on examination of fractal properties of signals and compare values of FD with reflectograms having always in mind definition of texture.

There is variety of examples of using fractal properties in literature such as with biomedical signals [4]–[6], seismograms [7], in telecommunication traffic analysis [8], and many other areas. Most often parameters used are multifractal spectrum (MFS) and FD obtained with different methods. MFS and its parameters are usually investigated for classification of signals, while FD is used for recognition of some characteristics of the signal. Our concept developed in this paper relies on fractals' capability to grade the quality of RIR segments. Overall assessment of RIR signal would be done trough cost function including individual segment evaluation.

Paper is organized as follows. In the second chapter we describe four FDs that will be used for testing texture criteria of RIR. The third chapter deals with computer generated signals as a preparation or calibration of FD methods for texture evaluation. Conclusions made in this chapter will be used in qualitative analysis of real signals in fourth chapter. In conclusion we discuss obtained results and propose next steps in texture description.

II. Fractal Dimension Algorithms

A. Variance Fractal Dimension Trajectory

For signals represented as fractional Brownian motion (fBm) process, $B_H(t)$, it is proved [3]:

$$Var\{\Delta B_H\} = |\Delta t|^{2H}, \tag{1}$$

where $\Delta B_H = B_H(t_2) - B_H(t_1)$ and $\Delta t = t_2 - t_1$. Here H stands for Hurst exponent. From (1) it follows:

$$H = \lim_{\Delta t \to 0} \left\{ \frac{1}{2} \cdot \frac{log(Var\{\Delta B_H\})}{log(|\Delta t|)} \right\}. \tag{2}$$

Fractal dimension, D_σ, can be calculated using Hurst exponent and Euclidean dimension, E ($E = 1$ for waveforms):

$$D_\sigma = E + 1 - H. \tag{3}$$

From (2) and (3) it is possible to determine variance fractal dimension trajectory (VFDT). VFDT plots variance

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of fractal dimension in time. For the time series, $S = \{s(1), s(2), \dots, s(M)\}$, it is calculated for each window of $N(< M)$ samples shifting in time for $shift$ samples. In [9] it is shown that windows should overlap, $shift < N$, in order to obtain better performances of VFDT. Variance of increment process within the window is calculated for range of increments, $\Delta t_k = 2^k$ [10]. Hurst exponent can be determined as a slope in log-log plot of variance as it is shown in (1).

B. *Katz algorithm*

Katz [11] algorithm assumes signal as a sequence of pairs consisting of sample number and value of the waveform: $S = \{(1, s_1), (2, s_2), \dots, (M, s_M)\}$. For two points in the sequence, (i, s_i) and (j, s_j), Euclidean distance can be calculated as:

$$dist(s_i, s_j) = \sqrt{(i-j)^2 + (s_i - s_j)^2}. \qquad (4)$$

The length of the curve of waveform is then determined as a sum of Euclidean distances between successive points:

$$L = \sum_{i=1}^{M-1} dist(i, i+1) \,. \qquad (5)$$

Katz defined fractal dimension as a consequence of exponential behavior in $L(d)$ graph:

$$D = \frac{\log(L)}{\log(d)} \qquad (6)$$

where $d = max\{dist(s_i, s_j)\}, i, j = 1, \dots, M$ is diameter (or planar extent) of the curve. If there are no intersections of the curve, d can be estimated as a maximum of distances between the first sample and all subsequent samples in time series.

Katz further normalized both length of the curve and diameter dividing them by average distance between successive points, $\bar{a}$:

$$D = \frac{\log(L/\bar{a})}{\log(d/\bar{a})} = \frac{\log(n)}{\log(n) + \log(d/L)}, \; n \equiv \frac{L}{\bar{a}} \qquad (7)$$

As it was earlier explained with VFDT, Katz algorithm is applied on overlapping windows [12] and given dependence fractal dimension – number of samples, i.e. fractogram, is further analyzed.

C. *Higuchi algorithm*

Higuchi algorithm [13] considers time series represented as: $S = \{s(1), \; s(2), \dots, s(M)\}$. From this time series we construct new ones:

$$S_k^m = \left\{ \begin{matrix} s(m), s(m+k), s(m+2k), \dots, \\ s\left(m + \left[\frac{M-m}{k}\right] \cdot k\right) \end{matrix} \right\}, \qquad (8)$$

$$m = 1, 2, \dots, k.$$

For each of such new time series, length of the curve is defined as:

$$L_m(k) = \frac{1}{k} \left\{ \sum_{i=1}^{\left[\frac{M-m}{k}\right]} |s(m+ik) - s(m+(i-1)k)| \cdot \frac{M-1}{\left[\frac{M-m}{k}\right] k} \right\} \qquad (9)$$

Length of the curve for the time interval k is then computed as the sum:

$$L(k) = \sum_{m=1}^{k} L_m(k) \,. \qquad (10)$$

Fractional dimension describes linear dependence in the double logarithmic scale graph of the length, $L(k)$:

$$L(k) = C \cdot k^{-D} \qquad (11)$$

where C is the constant. For different values for k, $L(k)$ is plotted and the dimension is calculated as a coefficient of linear regression using least-square method.

D. *Sevcik algorithm*

Sevcik [14] proposed method for calculating fractal dimension based on Hausdorff dimension:

$$D_h = \lim_{\epsilon \to 0} \frac{-log(N(\epsilon))}{log(\epsilon)}, \qquad (12)$$

where $N(\epsilon)$ is the number of segments of length 2ϵ needed to cover the curve with length L . Substituting N in (12) with $L/(2\epsilon)$ gives:

$$D_h = \lim_{\epsilon \to 0} \frac{-log(L) + \ln(2\epsilon)}{log(\epsilon)} = \lim_{\epsilon \to 0} \left(1 - \frac{\log(L) - \log(2)}{\log(\epsilon)}\right) \,. \qquad (13)$$

Length of the curve should be calculated according to (5). Since points of abscissa (usually in time units) and points of ordinate (e.g. amplitude units) have different units, it is questionable the unit of the length. Therefore Sevcik proposed normalization of points of both coordinates of the curve $S = \{(1, s_1), \dots, (i, s_i) \dots, (M, s_M)\}$:

$$i^* = \frac{i}{M} \,, \qquad (14a)$$

and

$$s_i^{\,*} = \frac{s_i - s_{min}}{s_{max} - s_{min}} \,. \qquad (14b)$$

After transformations in (14), the length of the curve is calculated using (5). If we assume $2\epsilon = 1/(M-1)$, fractal dimension becomes:

$$D = D_h = \lim_{M' \to \infty} \left(1 + \frac{\log(L)}{\log(2M')}\right), \; M' = M - 1. \qquad (15)$$

III. Computer Generated Signals

Definition of texture given in introduction will be analyzed here in means of fractograms calculated for synthetic signals reflecting all three criteria of texture. We consider *large number of early reflections* as Criterion 1, *no dominating reflections* as Criterion 2 and *uniformly/precisely spaced reflections* as Criterion 3. Signals are generated using Matlab:

Criterion 1:

```
S1 = zeros(1,1000);
S2 = S1;
q = 10*rand(1,250)+1;
S1(1:4:end) = q;
S2 (1:25:end) = q(1:40);
S = [S1 S2 S1 S2];
```

Criterion 2:

```
S1 = zeros(1,1000);
S2 = S1;
q = 10*rand(1,250)+1;
```

```
S1(1:4:end) = q;
S2 = S1/7;
S = [S1 S2 S1 S2];
```

Criterion 3:

```
S1 = zeros(1,1000);
q = 10*rand(1,250)+1;
S1(1:4:end) = q;
S2(randperm(1000)) = S1;
S = [S1 S2 S1 S2];
```

Here command *rand(m,n)* generates matrix of size mxn with values from the uniform distribution on the interval [0,1] and *randperm(n)* gives random permutation of integers 1,2,…,n. Results are presented in Fig.1. There are three signals describing all three criteria and computed fractograms for three methods: Katz, Higuchi and Sevcik. Length of the window for fractal dimension calculation is 100 samples and shifting value is 50 samples (50% overlapping). Comments on results in Fig. 1. will be given in the next chapter, together with results of real signal analysis.

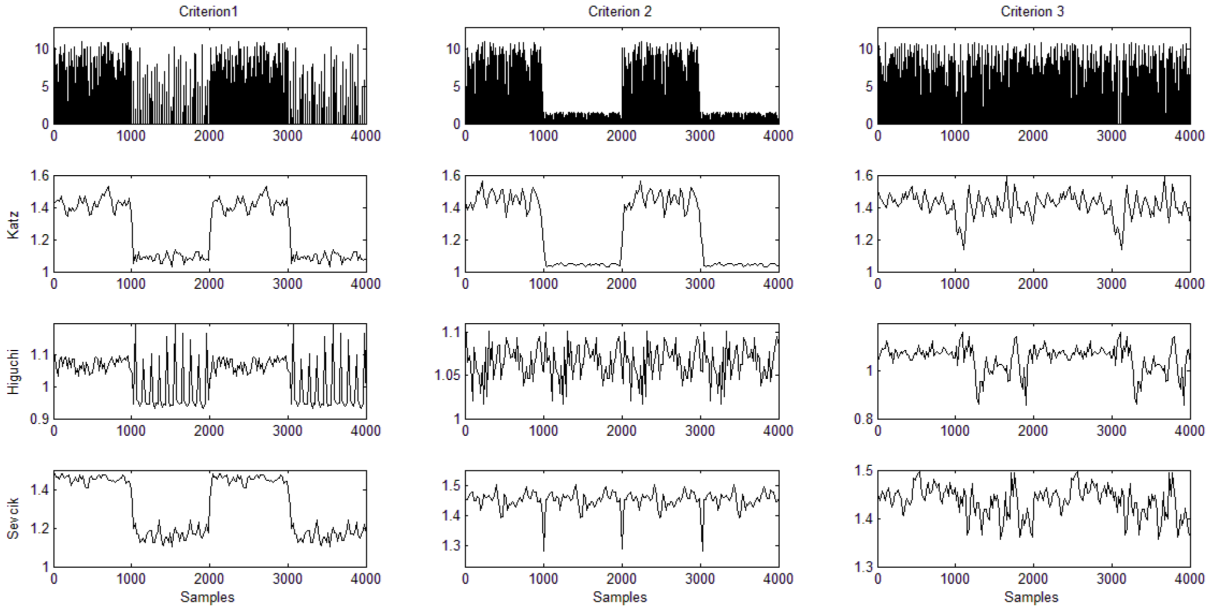

Fig. 1. Computer generated signals according texture criteria (first row), Katz's fractogram (second row), Higuchi's fractogram (third row) and Sevcik's fractogram (fourth row).

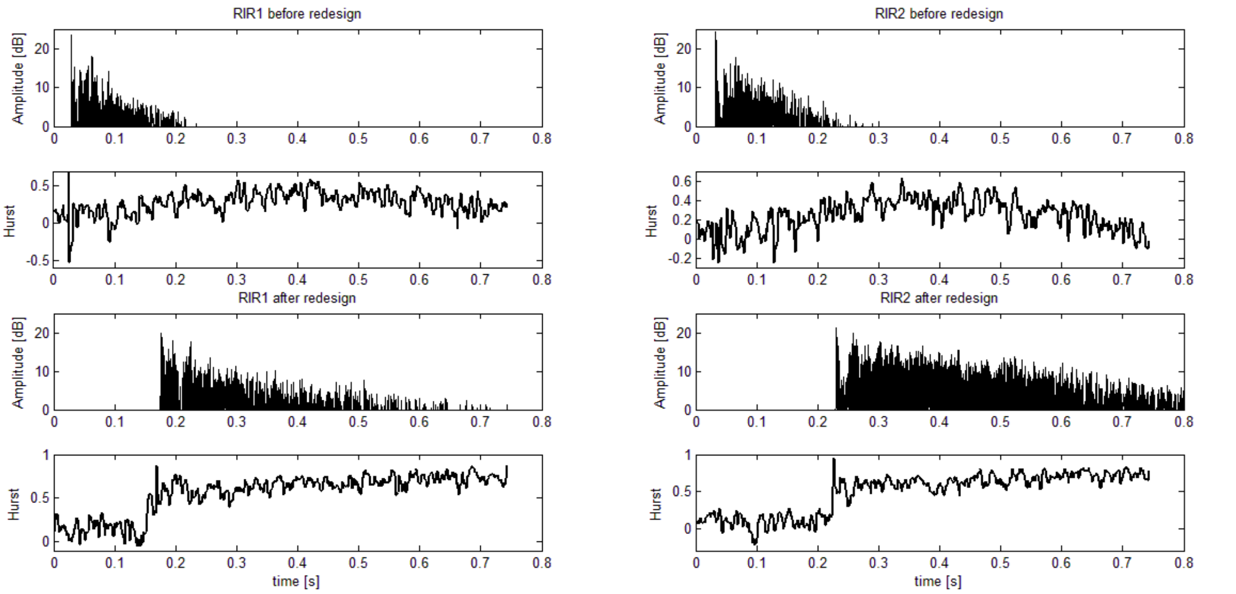

Fig. 2. Two different RIRs before acoustic redesign (first row) and corresponding Hurst exponents' variations (second row), RIRs after acoustic redesign (third row) and corresponding Hurst exponents (fourth row).

IV. REAL SIGNALS

We tested VFDT algorithm for two real RIRs as it is shown in Fig. 2. both in cases before and after acoustic room redesign. Sampling frequency is 44.1 kHz, length of window is 200 samples and shift is 50 samples. All signals are normalized in range [0,1] as it is important to make comparative analysis. From time domain it is obvious that RIRs after acoustic redesign are richer in number of reflections. It reflects on the Hurst exponent graph in higher values of this parameter. This is explained by longer-range dependence reached in the RIRs recorded after redesign comparing to RIRs recorded before redesign. Values of Hurst exponent around zero indicate short-range dependences (SRD).

Results of testing other three algorithms for FD calculation are presented in Fig. 3. It is seen, as it is expected after synthetic signal analysis, that Katz's algorithm gives excellent results for criteria 1 and 2. Wherever there are reflections missing in the signal it is reflected on Katz's fractogram corresponding to lower values of FD. It suggests us to find threshold in order to declare segments of the RIR above this threshold as criteria 1 and 2 are satisfied then. Higuchi's fractograms reflect changes of RIR segments quality according to criterion 1 in bigger variance, while we could not test its capabilities as indicator of criterion 3 due to nonzero values of samples which do not represent reflections. Criterion 1 therefore can be quantitatively described by thresholding the first derivative of the Higuchi's dimension. Finally, Sevcik's fractogram, as it is seen in Fig. 1. and Fig. 3., can be used for segmentation of regions in signal satisfying criteria 1 and 2. Actually, this method indicates changes in reflections' density (criterion 1) as fluctuations of FD's values. Also Sevcik's FD shows peaks where amplitude changes its value and it can be observed both in real and synthesized signals.

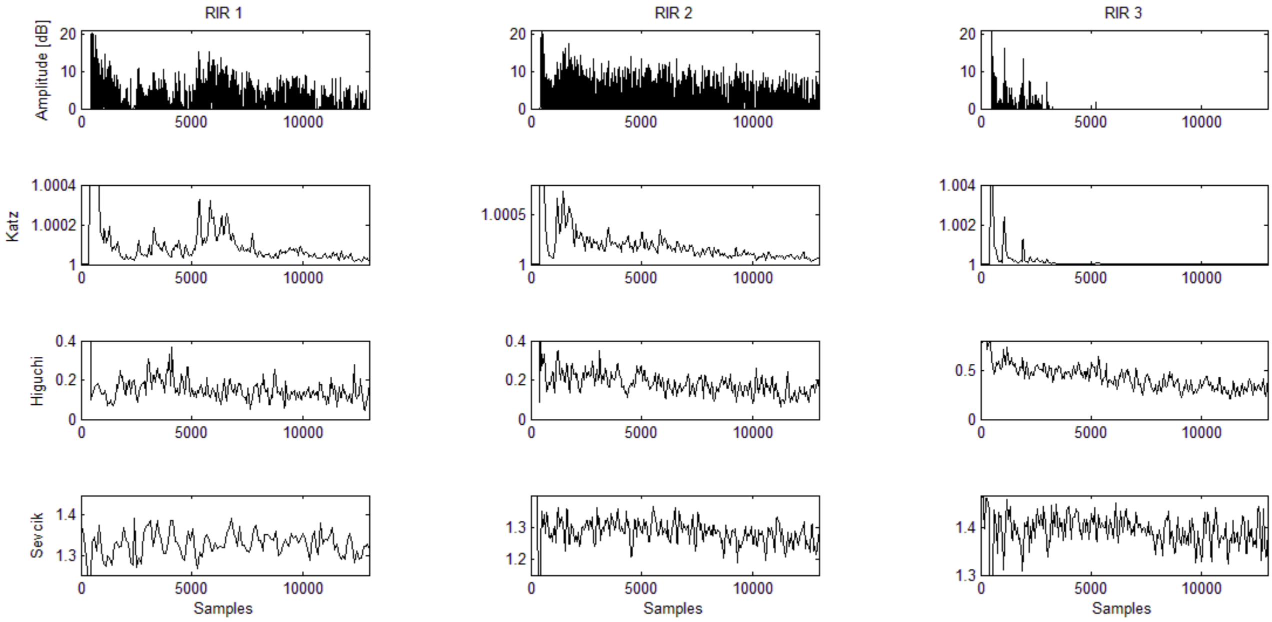

Fig. 3. Three different RIRs (first row) and corresponding fractograms: Katz's (second row), Higuchi's (third row) and Sevcik's (fourth row).

V. Conclusion

Results obtained in this paper imply the role of FD in classification of RIR signals in respect of texture definition. Thus, three algorithms (Katz, Higuchi and Sevcik) described in paper indicate changes in reflections density within reflectogram (criterion 1). Amplitude oscillations (criterion 2) can be detected in Katz's and Sevcik's fractograms, while uniformity of reflections disposition is evident in Higuchi's. It is also shown the influence of acoustical redesign of the hall on prolonging dependances in the signal (Hurst parameter greater than 0.5). In future we will continue our work on fractal analysis of RIR and seek for quantitative measures of texture quality, such as thresholding already discussed here.

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