Direction of Arrival Estimation using MUSIC and Root – MUSIC Algorithm

Andy Vesa

Abstract **— The resolution of a signal direction of arrival (DoA) estimation can be enhanced by an array antenna system with innovative signal processing. Super resolution algorithms take advantage of array structures to better process the incoming signals. They also have the ability to identify multiple targets. This paper explores the eigen-analysis category of super resolution algorithm. The performances of the MUSIC (Multiple Signal Classification) and the Root-MUSIC algorithm are presented in this paper.**

Keywords **— array antenna, direction of arrival estimation, MUSIC algorithm, Root-MUSIC algorithm.**

I. Introduction

THE need for Direction-of-Arrival estimation arises in many engineering applications including wireless communications, radar, radio astronomy, sonar, navigation, tracking of various objects, rescue and other emergency assistance devices. In its modern version, DoA estimation is usually studied as part of the more general field of array processing. Much of the work in this field, especially in earlier days, focused on radio direction finding – that is, estimating the direction of electromagnetic waves impinging on one or more antennas [1].

Over the last decade, Wireless Local Area Networks (WLANs) have received increased popularity due to their flexibility and convenience. A high-speed data rate is necessary in order to comply with the requirements of advanced services, such as internet broadcasting and conferencing. Due to the increasing over usage of the low end of the spectrum, people started to explore the higher frequency band for these applications, where more spectrum is available. With higher frequencies, higher data rate and higher user density, multipath fading and cross-interference become more serious issues, resulting in the degradation of bit error rate (BER). To combat these problems and to achieve higher communication capacity, smart antenna systems with adaptive beamforming capability have proven to be very effective in suppression of the interference and multipath signals [2].

Signal processing aspects of smart antenna systems has concentrated on the development of efficient algorithms for Direction-of-Arrival (DoA) estimation and adaptive beamforming. The recent trends of adaptive beamforming drive the development of digital beamforming systems [3].

DoA estimation using a fixed antenna has many limitation. Antenna main-lobe beamwidth is inversely proportional to its physical size. Improving the accuracy of angle measurement by increasing the physical aperture of the receiving antenna is not always a practical option. Certain systems such as a missile seeker or aircraft antenna have physical size limitations: therefore they have relatively wide main-lobe beamwidth. Consequently, the resolution is quite poor and if there are multiple signals falling in the antenna main-lobe, it is difficult to distinguish between them.

Instead of using a single antenna, an array antenna system with innovative signal processing can enhance the resolution of a DoA estimation. An array sensor system has multiple sensors distributed in space. This array configuration provides spatial samplings of the received waveform. A sensor array has better performance than the single sensor in signal reception and parameter estimation [4].

There are many different super resolution algorithms including spectral estimation, model based, eigen-analysis. The various DoA estimation algorithms are Bartlett, Capon, Min-norm, MUSIC and ESPRIT. The MUSIC algorithm is one of the most popular and widely used subspace-based techniques for estimating the DoAs of multiple signal sources. The conventional MUSIC algorithm involves a computationally demanding spectral search over the angle and, therefore, its implementation can be prohibitively expensive in real-world applications. The Root-MUSIC algorithm enjoys a substantially reduced computational complexity and an improve threshold estimation performance as compared to the spectral MUSIC approach, it is only applicable to uniform linear arrays (ULA) or non-uniform linear arrays whose sensors are restricted to lie on a uniform grid [5].

In this paper, a computer simulation programs using Matlab were developed to evaluate the direction-of-arrival performance of MUSIC and Root-MUSIC algorithms.

II. MUSIC Algorithm

In wireless transmission, the receiving antennas can collect more signals that can be emitted by several sources, as shown in Fig. 1. An important fact is the direction of arrival estimation of signals received from different sources.

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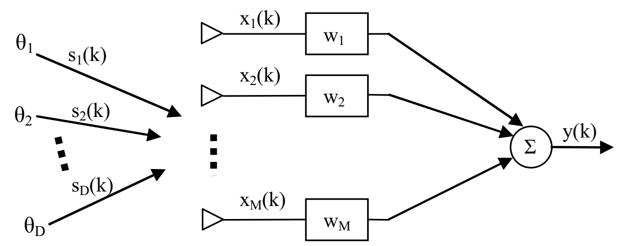


Fig. 1. The Uniform Linear Array (ULA).

It can be observed that the D signals arrive from D directions. They are received by an array of M elements with M potential weights.

Many of the DoA algorithms rely on the array correlation matrix. In order to simplify the notation let us define the M x M array correlation matrix R_{xx} as [6]:

$$\begin{aligned} R_{xx} &= E\left[\bar{x}\cdot\bar{x}^{H}\right] = E\left[\left(\bar{A}\bar{s}+\bar{n}\right)\left(\bar{s}^{H}\bar{A}^{H}+\bar{n}^{H}\right)\right] \\ &= \bar{A}E\left[\bar{s}\cdot\bar{s}^{H}\right]\bar{A}^{H} + E\left[\bar{n}\cdot\bar{n}^{H}\right] \\ &= \bar{A}R_{ss}\bar{A}^{H} + R_{nn}. \end{aligned} \quad (1)$$

where R_{SS} represents the source correlation matrix (D x D elements), $R_{nn} = \sigma_n^2 I$ represents the noise correlation matrix (M x M elements), and I represents the identity matrix (N x N elements).

Given M-array elements with D-narrowband signal sources and uncorrelated noise, we can make some assumptions about the properties of the correlation matrix: is an MxM Hermitian matrix. The array correlation matrix has M eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_M$) along with associated eigenvectors $\bar{E} = \left[\bar{e}_1 \bar{e}_2 \cdots \bar{e}_M\right]$. If the eigenvalues are sorted from smallest to largest, we can divide the matrix $\bar{E}$ into two subspaces: $\bar{E} = \left[\bar{E}_N \bar{E}_S\right]$. The first subspace $\bar{E}_N$ is called the noise subspace and is composed of $M - D$ eigenvectors associated with the noise, and the second subspace $\bar{E}_S$ is called the signal subspace and is composed of D eigenvectors associated with the arriving signals. The noise subspace is an M x $(M - D)$ matrix, and the signal subspace is an $(M$ x $D)$ matrix.

The MUSIC algorithm is based on the assumption that the noise subspace eigenvectors are orthogonal to the array steering vectors, $\bar{a}(\theta)$, at the angles of arrival $\theta_1, \theta_2, \ldots, \theta_D$. Because of this orthogonality condition, one can show that the Euclidian distance $d^2 = \bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{a}(\theta) = 0$ for each and every arrival angle $\theta_1, \theta_2, \ldots, \theta_D$. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudospectrum is:

$$P_{MUSIC}(\theta) = \frac{1}{\bar{a}(\theta)^H \bar{E}_N \bar{E}_N^H \bar{a}(\theta)}. \quad (2)$$

III. ROOT-MUSIC ALGORITHM

The MUSIC algorithm in general can apply to any arbitrary array regardless of the position of the array elements. Root-MUSIC implies that the MUSIC algorithm is reduced to finding roots of a polynomial as opposed to merely plotting the pseudospectrum or searching for peaks in the pseudospectrum.

One can simplify the denominator in the expression (2) by defining the matrix $\bar{C} = \bar{E}_N \bar{E}_N^H$ which is Hermitian. This leads to the root-MUSIC expression:

$$P_{RootMUSIC}(\theta) = \frac{1}{\left|\bar{a}(\theta)^{\theta}\,\bar{C}\,\bar{a}(\theta)\right|}. \quad (3)$$

If we have an ULA, the m-th element of the array steering vector is given by:

$$a_m(\theta) = e^{jkd(m-1)\sin\theta} \qquad m = 1, 2, \ldots, M. \quad (4)$$

The denominator argument in (3) can be written as:

$$\begin{aligned} \bar{a}(\theta)^H\,\bar{C}\,\bar{a}(\theta) &= \sum_{m=1}^{M}\sum_{n=1}^{M} e^{-jkd(m-1)\sin\theta}\, C_{mn}\, e^{jkd(n-1)\sin\theta} \\ &= \sum_{l=-M+1}^{M-1} c_l e^{jkdl\sin\theta}. \end{aligned} \quad (5)$$

where c_l is the sum of the diagonal elements of $\bar{C}$ has off-diagonal sums such that $c_0 > |c_l|$ for $l \neq 0$. Thus the sum of off-diagonal elements is always less than the sum of the main diagonal elements. In addition, $c_l = c_{-l}^*$.

We can simplify (5) to be in the form of a polynomial whose coefficients are c_l. Thus,

$$D(z) = \sum_{l=-M+1}^{M-1} c_l z^l. \quad (6)$$

where $z = e^{-jkd\sin\theta}$.

The roots of $D(z)$ that lie closest to the unit circle correspond to the poles of the MUSIC pseudospectrum. Thus, this technique is called *Root-MUSIC.* The polynomial of (5) is of order $2(M-1)$ and thus has roots of $z_1, z_2, \ldots, z_{2(M-1)}$. Each root can be complex and using polar notation can be written as:

$$z_i = |z_i| e^{j\arg(z_i)} \qquad i = 1, 2, \ldots, 2(M-1). \quad (7)$$

where $\arg(z_i)$ is the phase angle of z_i.

Exact *zeros* in $D(z)$ exist when the root magnitudes $|z_i| = 1$. One can calculate the DoA by comparing $e^{j\arg(z_i)}$ to $e^{-jkd\sin\theta}$ to get:

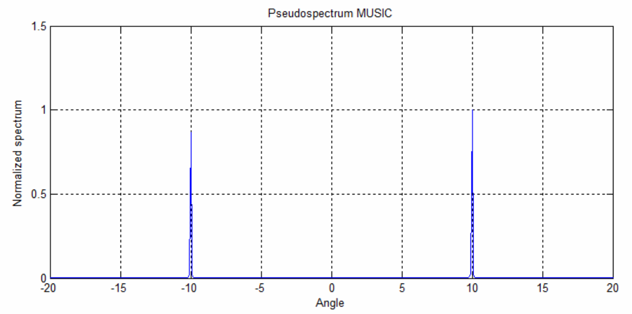
$$\theta_i = -\sin^{-1}\left(\frac{1}{kd}\arg(z_i)\right). \quad (8)$$

IV. SIMULATION RESULTS

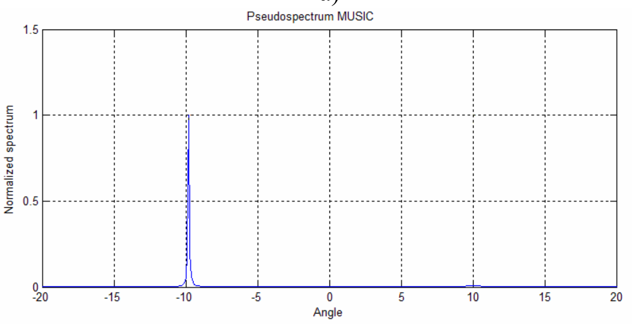
In this section, computer simulations are provided to substantiate the performance analysis. In all cases, the impinging angles of the sources are relative to the broadside of a linear uniform array. The space between two adjacent array elements is one half of a wavelength. The additive background noise is assumed to be spatially and temporally white complex Gaussian with zero – mean.

In these simulations, it is considered a linear array antenna formed by 4 elements that are evenly spaced with the distance of $\lambda/2$. The noise is considered to be additive, having the 0.1 variance value. It is considered that two signals are emitted from two sources placed on the directions $\theta_1 = -10°$ and, respectively $\theta_2 = +10°$. These two signals are considered to have equal amplitudes.

In figure 2 pseudospectrum obtained by MUSIC algorithm for SNR= 10 dB and, respectively, SNR= -5 dB is presented.



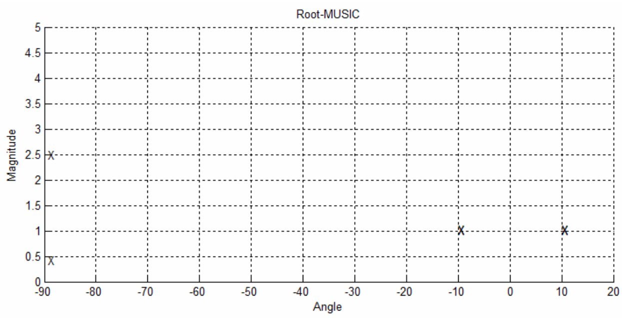
a)



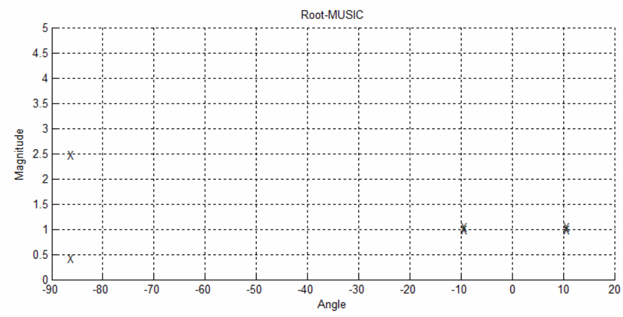
b)

Fig. 2. MUSIC pseudospectrum for SNR= 10 dB (a) and SNR= -5 dB (b).

For this pseudospectrum, we can find the roots and then solve for magnitude and angles of the six roots, which are presented in figure 3, for SNR= 10 dB and, respectively, SNR= -5 dB.



a)



b)

Fig. 3. The magnitude and angles of the 6 roots obtained using Root-MUSIC algorithm for SNR= 10 dB (a) and SNR= -5 dB (b).

In figure 4 the location of all 6 roots in cartesian coordinates is presented. It is clear that only the four on the right side of 0 are nearest to the unit circle and are close to the expected angles of arrival.

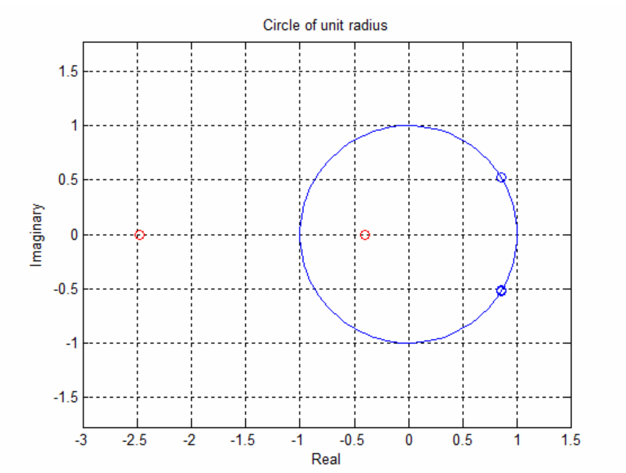


Fig. 4. All six roots in cartesian coordinates.

In figure 5 the MUSIC pseudospectrum and roots found with root-MUSIC, obtained for SNR= -5 dB is presented.

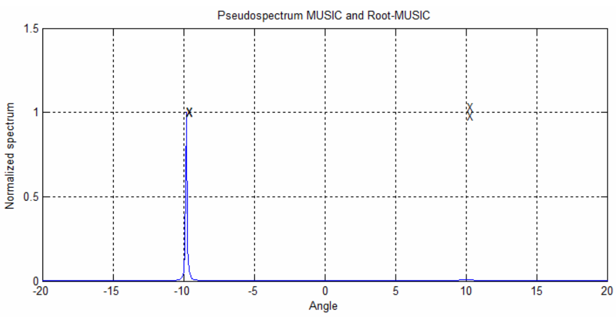


Fig. 5. MUSIC pseudospectrum and roots found with root-MUSIC for SNR= -5 dB.

The roots found with root-MUSIC earlier do not exactly reflect the actual location of the angles of arrival of $\theta_1 = -10^\circ$ and $\theta_2 = +10^\circ$ but they do indicate two angles of arrival. The root themselves show the existence of an angle of arrival at near $+10^\circ$ which is not obvious from the plot of the MUSIC pseudospectrum. The error in locating the correct root locations owes to the fact that the incoming signals are partially correlated, that we approximated the correlation matrix by time averaging, and that the SNR is relatively low.

V. CONCLUSION

In this paper, I give extensive computer simulation results to demonstrate the performances of the MUSIC and Root-MUSIC algorithm. The MUSIC and Root-MUSIC algorithm are based on the eigen-analysis of the array correlation matrix.

It is observed that the Root-MUSIC algorithm is not so powerful, but in some cases the results obtained with this algorithm are acceptable. These two algorithms enhance the DoA estimation. With use of MUSIC algorithms smart antennas add a new possibility of user separation by space through Space Division Multiple Access (SDMA).

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