Effects of Imperfect Reference Signal Recovery on BER Performance of SC Receiver over Composite Fading Channel

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Abstract **— In this paper we discuss a detection of quadrature phase-shift keying (QPSK) signals in a generalized *K* composite fading channel. The selection combining is applied at the reception, while the branches of the combiner are not necessarily identically distributed. The imperfect carrier extraction from the non-modulated pilot signal is taken into account through the phase error that occurs in phase-locked loop (PLL) circuit and it follows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance is examined.**

Keywords **— Composite fading channel, diversity, imperfect carrier extraction, selection combining, Tikhonov distribution.**

I. Introduction

In mobile radio communications, due to a multipath propagation, the incoming signal at the receiver is corrupted by the fast fading effect i.e. the random fast fluctuations of the signal envelope. Also, due to the nature of the propagation medium, there can be also random slow fluctuations of the received average signal power (shadowing effect) [1]. In certain propagation environments the simultaneous influence of both fast and slow fading effect must be taken into account representing a propagation channel by a composite fading model. A general composite model assumes Nakagami-*m* distribution of signal envelope and lognormal distribution of average signal power [2]. However, a composite probability density function, obtained in this way, is in integral form and it is not convenient for further analysis. For this reason, the equivalent gamma distribution is suggested in the literature for describing slow fading effect, showing very good match to the experimental results [3]-[6]. The obtained composite probability density function follows in this case a generalized (K_G) distribution [6].

Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading and increasing the communication reliability without enlarging either transmitting power or bandwidth of the channel. Particular diversity methods and combining techniques are presented in [1]. Selection combining (SC) is combining technique where the strongest signal is chosen among L branches of diversity system [1]. Although SC technique brings the smallest improvement of receiver performances, the simplicity of practical realization makes the mentioned technique widely spread.

The phase-locked loop (PLL) is used for carrier signal recovery from non-modulated pilot signal in the receiver. As the receiver is not ideal, a certain phase error appears. In [6] a detailed performance analysis for the most important diversity receivers (including SC), operating over a composite fading channel, was presented. Expressions for important statistical metrics have been derived. However, no phase error during extraction of the reference carrier was considered. The phase error is a difference between the phase of the incoming signal and the phase of the recovered carrier signal in the loop, and this may lead to serious degradation to system performance. It is a statistical process which follows Tikhonov distribution [7].

In this paper we discuss the detection of quadrature phase shift keying (QPSK) signals in a composite fading channel, which follows the K_G distribution. The selection combining is applied at the reception, while the branches of the combiner in general are not identically distributed. The imperfect carrier signal recovery from the non-modulated signal is taken into account through the phase error that occurs in PLL circuit. It is a random process and follows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance is examined.

II. System and Channel Model

After the propagation through the composite fading channel, signal at the k-th branch of SC receiver has the

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$$z_k(t) = r_k(t)\cos(\omega_0 t + \Phi_0 + \delta_k(t)) + n_k(t)\,, \qquad (1)$$

where $r_k(t)$ is the envelope of the received signal, ω_0 is the angular frequency of the carrier, Φ_0 is the transmitted phase of the signal, $\delta_k(t)$ is the random phase (the phase noise caused by a fading), and $n_k(t)$ is the additive white Gaussian noise (AWGN) in the k-th diversity branch with zero mean value and variance σ^2. It is assumed that the noise power is the same in every diversity branch and fading is uncorrelated among different branches. Depending on a sent symbol, in the case of QPSK signal detection, Φ_0 can take following values from the set $\Phi_0 \in \{\pi/4, 3\pi/4, -3\pi/4, -\pi/4\}$.

Since the signal is transmitted over a composite fading channel, envelope of the signal in k-th input branch, $r_k(t)$, is a statistical process and its instantaneous values fallow K_G distribution

$$p_k(r_k) = \frac{4}{\Gamma(m_m)\Gamma(m_s)}\left(\frac{m_m m_s}{\Omega_{sk}}\right)^{\frac{m_m+m_s}{2}} r_k^{m_m+m_s-1} \times \\ \times K_{m_s-m_m}\left(2r_k\sqrt{\frac{m_m m_s}{\Omega_{sk}}}\right), \quad r_k > 0 \qquad (2)$$

where the second kind modified Bessel function of order v is denoted by $K_v(\cdot)$ [8, Eqn. (8.432)] and $\Gamma(.)$ is gamma function [8, Eqn. (8.310)]. Parameter m_m is fast Nakagami-m fading parameter and it can take values from the range $0.5 \le m_m < \infty$. Larger values of this parameter indicate a smaller fading severity. Parameter m_s is a shadowing parameter and its larger values are related to the smaller shadowing. The average signal power in k-th input branch is $\overline{r_k^2} = \int_0^\infty r_k^2 p_k(r_k) dr_k = \Omega_{sk}$. It can be shown that probability density function (PDF) of instantaneous signal-to-noise ratio (SNR) is then

$$p_k(\rho_k) = \frac{2}{\Gamma(m_m)\Gamma(m_s)}\left(\frac{m_m m_s}{\rho_{0k}}\right)^{\frac{m_m+m_s}{2}} \rho_k^{\frac{m_m+m_s}{2}-1} \times \\ \times K_{m_s-m_m}\left(2\sqrt{\rho_k \frac{m_m m_s}{\rho_{0k}}}\right), \quad \rho_k \ge 0 \qquad (3)$$

where ρ_{0k} is average symbol SNR in k-th branch. The relation between the average symbol and bit SNR is $\rho_{0k} = \rho_{0bk}\log_2 M$, where M is the number of modulation levels (in the case of QPSK detection it is M=4), $\log_2(\cdot)$ is the logarithm to base 2 and ρ_{0bk} is average bit SNR.

The chosen branch in SC circuit is the one with the strongest signal. The PDF of the SNR at the output of the combining circuit with L non identical branches can be written as [1]:

$$p_\rho(\rho) = \sum_{k=1}^{L}\left(p_k(\rho_k)\prod_{\substack{i=1 \\ i\ne k}}^{L} f_i(\rho_i)\right) \qquad (4)$$

where $p_k(\rho)$ is the PDF of instantaneous SNR at the k-th branch and $f_i(\rho_i)$ the cumulative distribution function (CDF) at the i-th branch, defined as

$$f_i(\rho_i) = \int_0^{\rho_i} p_i(t)dt\,. \qquad (5)$$

In the special case of independent and identically distributed (iid) branches expression (4) becomes

$$p_\rho(\rho) = L \cdot p_k(\rho_k) f_k^{L-1}(\rho_k)\,. \qquad (6)$$

The purpose of the PLL is to estimate the phase of the incoming signal. In ideal case, the estimated phase should be equal to the phase of the incoming signal $\delta_i(t)$. However, in practical realizations there is certain disagreement between the estimated phase $\hat{\delta}(t)$ and the phase of the signal $\delta_i(t)$. This disagreement is phase error and it is expressed as $\varphi(t) = \delta_i(t) - \hat{\delta}(t)$. The PDF for this phase error corresponds to Tikhonov distribution [7] and in the case of QPSK signal detection it is

$$p_\varphi(\varphi) = \frac{e^{\alpha_{PLL}\cdot\cos\varphi}}{2\pi \cdot I_0(\alpha_{PLL})}, \quad -\pi \le \varphi < \pi \qquad (7)$$

where the parameter α_{PLL} represents the signal-to-noise ratio in the PLL circuit and gives the information about the preciseness of phase estimation of incoming signal. It can be assumed $\alpha_{PLL}=1/\sigma_\varphi^2$, where σ_φ is a standard deviation of the phase error [7].

The expression for the conditional BER for QPSK signal, as a function of instantaneous symbol SNR in the channel $\rho = \frac{r^2}{2\sigma^2}$, $\sigma^2 = \overline{n^2(t)}$ and phase error φ, can be presented as

$$P_b(\varphi,\rho)_{QPSK} = \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{\rho}{\log_2 M}}(\cos\varphi - \sin\varphi)\right) + \\ + \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{\rho}{\log_2 M}}(\cos\varphi + \sin\varphi)\right) \qquad (8)$$

where $\mathrm{erfc}(\cdot)$ is the complementary error function [8, Eqn. (8.250)].

The average BER can be obtained by averaging (8) over all possible values of instantaneous symbol SNR ρ and phase error φ

$$P_{b|QPSK} = \int_0^{+\infty}\int_{-\pi}^{\pi} P_b(\varphi,\rho)_{|QPSK}\, p_\rho(\rho) p_\varphi(\varphi) d\varphi d\rho, \\ -\pi \le \varphi \le \pi. \qquad (9)$$

III. Numerical Results

Using (6)-(9), one can calculate the average BER for K_G fading channel and discuss performances of the receiver for different values of m_m and m_s parameters, standard deviation of phase noise, σ_φ, as well as for different number of diversity branches, L.

In Fig. 1 the influence of shadowing intensity (ms parameter) on BER of QPSK signal detection is presented for different values of fast fading parameter m_m. A selection combiner with two branches is used at the reception. Diversity branches are assumed non-identically distributed and fast fading parameters differ in each branch (in this case m_{m1} and m_{m2}). A phase error standard deviation is σ_φ. One can notice, regardless of the values of m_m and m_s parameters, a BER floor appears when it is σ_φ =10°. Therefore, no increase of ρ_{0b1} can cause the BER to fall under the certain value. For smaller values of m_m (deeper fast fading) BER floor earlier arises, i.e. for larger average bit BER values. Also, the influence of shadowing on BER is less emphasized in the case of larger fast fading intensity (small m_m).

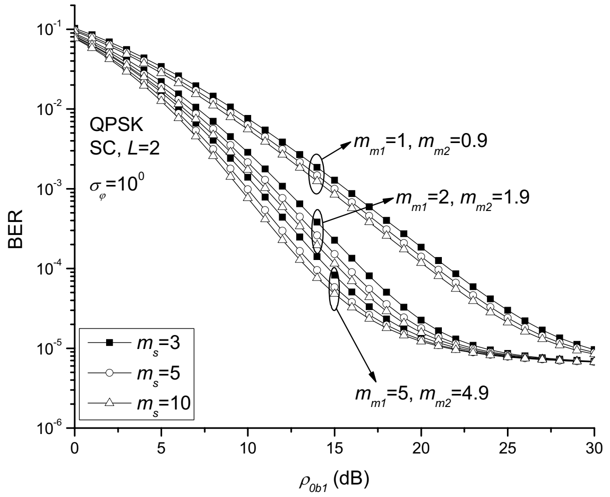

Fig. 1. Influence of fading and shadowing parameters on BER performance.

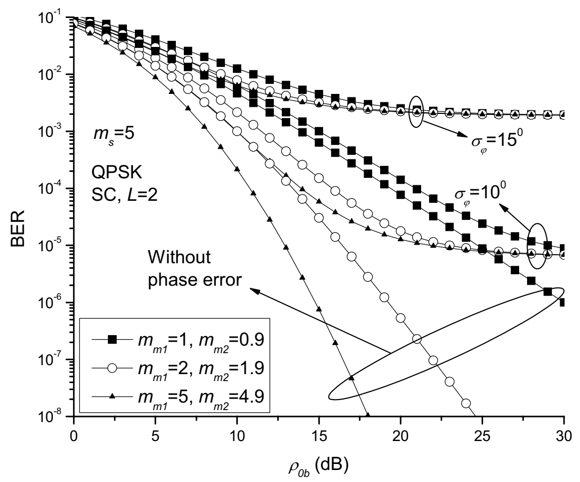

Fig. 2. Error probability for different values of phase error standard deviation.

Fig. 2 shows the influence of phase error on BER, for different values of fast fading parameter m_m. For the purpose of comparison, an ideal case, the one without phase error, is given, also. Obviously, the phase error extremely impairs system performance. The phase error of σ_φ=10° already brings the BER floor and the increase of ρ_{0b1} can not further improve quality of reception. Of course, with the increase of σ_φ, this BER floor appears for lower ρ_{0b1} values. It can be concluded that the quality of PLL circuit in the receiver has a crucial importance.

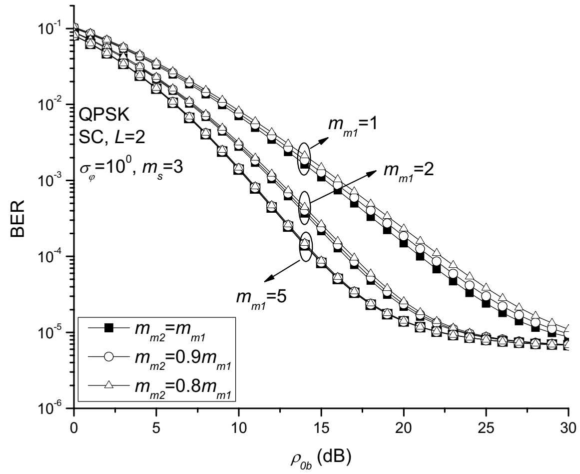

Fig. 3. Influence of non-identical fading distribution in diversity branches on system performance.

In Fig. 3 one can observe the impact of non-identical fading distribution in diversity branches on system performance. The case of dual branch diversity reception and the different values of fast fading parameters in the first branch, m_{m1}, is presented, while fading parameter of the second branch, m_{m2}, deviates by 10% and 20% of m_{m1} value. It can be seen that this effect of non-identically distributed branches achieves the greatest impact on the BER in the case of higher fast fading severity (smaller m_m).

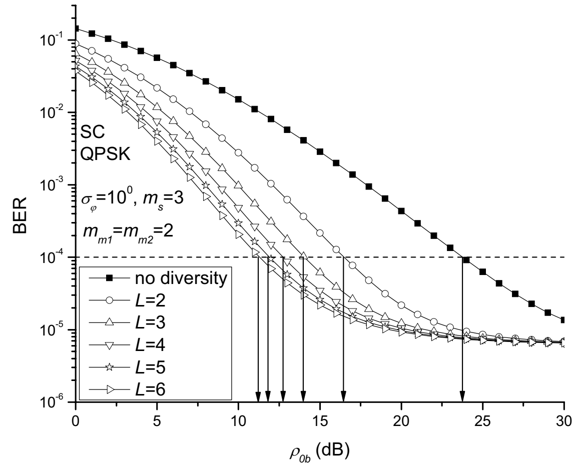

Fig. 4. Influence of diversity order on BER performance.

The influence of diversity order on the performances of the receiver can be observed from Fig. 4 where dependence of the average BER on average SNR (ρ_{0b1}) is shown for different values of parameter L. With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity. Power gain is the highest when order of diversity system changes from $L=1$ to $L=2$. For example, in order to obtain the same value of BER=10^{-4}, for parameter values m_{m1}=m_{m2}=2, m_s=3 and σ_φ=10°, it is necessary for average SNR to reach the value of

ρ_{0b1}=23.7dB for L=1, ρ_{0b1}=16.44dB for L=2, ρ_{0b1}=13.98dB for L=3, ρ_{0b1}=12.74dB for L=4, ρ_{0b1}=11.8dB for L=5, and ρ_{0b1}=11.2dB for L=6. It can be noticed that the gain exponentially declines with the increase of the order of diversity system.

Fig. 5 shows the influence of fading and shadowing parameters on minimum input average SNR (sensitivity) of the SC receiver required to produce a BER value of 10^{-4}. Branches of the receiver are assumed identically distributed. Curves corresponding to sensitivity of 15dB, 19dB and 22dB are presented for different values of phase error standard deviation σ_φ. The sensitivity increases with the decrease of σ_φ value. This effect is especially emphasized when shadowing and fading severity are low. For large σ_φ values the BER floor appears and it totally determines the sensitivity.

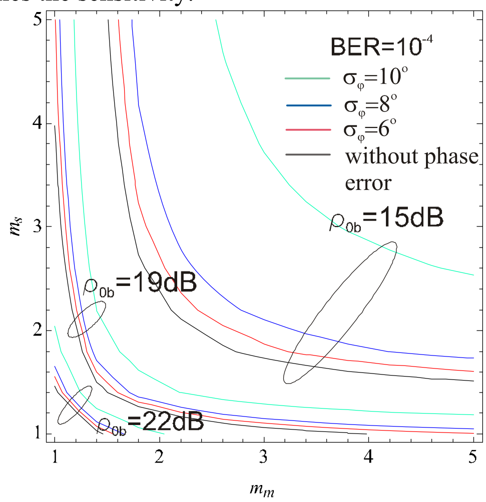

Fig. 5. Required values of average SNR in order to achieve error probability 10^{-4} for different values of phase error standard deviation σ_φ.

IV. Conclusion

In this paper we discussed the detection of QPSK signals in a composite fading channel. The selection combining is applied at the reception, while the branches of the combiner are not necessarily identically distributed. The imperfect carrier signal recovery is taken into account through the phase error that occurs in PLL circuit. It is a random process and fallows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance was examined.

As it was expected, BER decreases when shadowing and fading severity become low. The influence of shadowing parameter becomes determinative when the fast fading severity is low.

The phase error extremely impairs system performance. The phase error of σ_φ=10° already brings the BER floor and the increase of ρ_{0b1} can not further improve quality of reception. Of course, with the increase of σ_φ, this BER floor appears for lower ρ_{0b1} values.

The non-identically distributed branches of SC receiver impair the system performance. This effect achieves the greatest impact on the BER in the case of higher fast fading severity.

With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity.

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