

Effects of Imperfect Reference Signal Recovery on BER Performance of SC Receiver over Composite Fading Channel

Bojana Z. Nikolić, Student Member, IEEE, Goran T. Đorđević, Member, IEEE, Dejan Milić, Member, IEEE, Mihajlo Č. Stefanović, Member, IEEE

Abstract — In this paper we discuss a detection of quadrature phase-shift keying (QPSK) signals in a generalized $K$ composite fading channel. The selection combining is applied at the reception, while the branches of the combiner are not necessarily identically distributed. The imperfect carrier extraction from the non-modulated pilot signal is taken into account through the phase error that occurs in phase-locked loop (PLL) circuit and it follows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance is examined.

Keywords — Composite fading channel, diversity, imperfect carrier extraction, selection combining, Tikhonov distribution.

I. INTRODUCTION

In mobile radio communications, due to a multipath propagation, the incoming signal at the receiver is corrupted by the fast fading effect i.e. the random fast fluctuations of the signal envelope. Also, due to the nature of the propagation medium, there can be also random slow fluctuations of the received average signal power (shadowing effect) [1]. In certain propagation environments the simultaneous influence of both fast and slow fading effect must be taken into account representing a propagation channel by a composite fading model. A general composite model assumes Nakagami-$m$ distribution of signal envelope and lognormal distribution of average signal power [2]. However, a composite probability density function, obtained in this way, is in integral form and it is not convenient for further analysis. For this reason, the equivalent gamma distribution is suggested in the literature for describing slow fading effect, showing very good match to the experimental results [3]-[6]. The obtained composite probability density function follows in this case a generalized ($K_{\alpha}$) distribution [6].

Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading and increasing the communication reliability without enlarging either transmitting power or bandwidth of the channel. Particular diversity methods and combining techniques are presented in [1]. Selection combining (SC) is combining technique where the strongest signal is chosen among $L$ branches of diversity system [1]. Although SC technique brings the smallest improvement of receiver performances, the simplicity of practical realization makes the mentioned technique widely spread.

The phase-locked loop (PLL) is used for carrier signal recovery from non-modulated pilot signal in the receiver. As the receiver is not ideal, a certain phase error appears. In [6] a detailed performance analysis for the most important diversity receivers (including SC), operating over a composite fading channel, was presented. Expressions for important statistical metrics have been derived. However, no phase error during extraction of the reference carrier was considered. The phase error is a difference between the phase of the incoming signal and the phase of the recovered carrier signal in the loop, and this may lead to serious degradation to system performance. It is a statistical process which follows Tikhonov distribution [7].

In this paper we discuss the detection of quadrature phase shift keying (QPSK) signals in a composite fading channel, which follows the $K_{\alpha}$ distribution. The selection combining is applied at the reception, while the branches of the combiner in general are not identically distributed. The imperfect carrier signal recovery from the non-modulated signal is taken into account through the phase error that occurs in PLL circuit. It is a random process and follows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance is examined.

II. SYSTEM AND CHANNEL MODEL

After the propagation through the composite fading channel, signal at the $k$-th branch of SC receiver has the
form
\[ z_k(t) = r_k(t) \cos(\omega_0 t + \Phi_0 + \delta_k(t)) + n_k(t), \]
where \( r_k(t) \) is the envelope of the received signal, \( \omega_0 \) is the angular frequency of the carrier, \( \Phi_0 \) is the transmitted phase of the signal, \( \delta_k(t) \) is the random phase (the phase noise caused by a fading), and \( n_k(t) \) is the additive white Gaussian noise (AWGN) in the \( k \)-th diversity branch with zero mean value and variance \( \sigma_n^2 \). It is assumed that the noise power is the same in every diversity branch and fading is uncorrelated among different branches. Depending on a sent symbol, in the case of QPSK signal detection, \( \Phi_0 \) can take following values from the set \( \Phi_0 \in \{\pi/4,3\pi/4,-3\pi/4,-\pi/4\} \).

Since the signal is transmitted over a composite fading channel, envelope of the signal in \( k \)-th input branch, \( r_k(t) \), is a statistical process and its instantaneous values follow \( K_G \) distribution
\[
p_k(r_k) = \frac{4}{\Gamma(m_k) \Gamma(m_s)} \left( \frac{m_mm_k}{\Omega_{sk}} \right)^{m_m+m_s} r_k^{m_m+m_s-1} \times K_{m_m-m_s} \left( 2r_k \sqrt{ \frac{m_mm_s}{\Omega_{sk}} } \right), \quad r_k > 0
\]
where the second kind modified Bessel function of order \( \nu \) is denoted by \( K_\nu(\cdot) \) [8, Eqn. (8.432)] and \( \Gamma(\cdot) \) is gamma function [8, Eqn. (8.310)]. Parameter \( m_m \) is fast Nakagami-m fading parameter and it can take values from the range \( 0.5 \leq m_m < \infty \). Larger values of this parameter indicate a smaller fading severity. Parameter \( m_s \) is a shadowing parameter and its larger values are related to the smaller shadowing. The average signal power in \( k \)-th input branch is
\[
\overline{r_k^2} = \int_0^\infty r_k^2 p_k(r_k) dr_k = \Omega_{sk}.
\]
It can be shown that probability density function (PDF) of instantaneous signal-to-noise ratio (SNR) is then
\[
p_k(\rho_k) = \frac{2}{\Gamma(m_k) \Gamma(m_s)} \left( \frac{m_mm_k}{\rho_{0k}} \right)^{m_m+m_s} \rho_k^{m_m+m_s-1} \times K_{m_m-m_s} \left( 2\sqrt{ \frac{m_mm_s}{\rho_{0k}} } \rho_k \right), \quad \rho_k \geq 0
\]
where \( \rho_{0k} \) is average symbol SNR in \( k \)-th branch. The relation between the average symbol and bit SNR is \( \rho_{0k}=\rho_{0sk}\log_2 M \), where \( M \) is the number of modulation levels (in the case of QPSK detection it is \( M=4 \)). \( \log_2(\cdot) \) is the logarithm to base 2 and \( \rho_{0sk} \) is average bit SNR.

The chosen branch in SC circuit is the one with the strongest signal. The PDF of the SNR at the output of the combining circuit with \( L \) non identical branches can be written as [1]:
\[
P_{\rho}(\rho) = \sum_{k=1}^{L} \left[ p_k(\rho_k) \prod_{i=1}^{L} f_i(\rho_i) \right]
\]
where \( p_k(\rho_k) \) is the PDF of instantaneous SNR at the \( k \)-th branch and \( f_i(\rho_i) \) is the cumulative distribution function (CDF) at the \( i \)-th branch, defined as
\[
f_i(\rho_i) = \int_0^{\rho_i} p_i(t) dt.
\]
In the special case of independent and identically distributed (iid) branches expression (4) becomes
\[
p_{\rho}(\rho) = L \cdot p_k(\rho_k) \int_0^{\rho_k} (\rho_k) dt.
\]

The purpose of the PLL is to estimate the phase of the incoming signal. In ideal case, the estimated phase should be equal to the phase of the incoming signal \( \delta(t) \). However, in practical realizations there is certain disagreement between the estimated phase \( \hat{\delta}(t) \) and the phase of the signal \( \delta(t) \). This disagreement is phase error and it is expressed as \( \epsilon(t) = \delta(t) - \hat{\delta}(t) \). The PDF for this phase error corresponds to Tikhonov distribution [7] and in the case of QPSK signal detection it is
\[
p_{\epsilon}(\epsilon) = \frac{e^{\epsilon_0 \rho_{PLL}}} {2 \pi \sigma_\epsilon^2}, \quad -\pi \leq \epsilon < \pi
\]
where the parameter \( \rho_{PLL} \) represents the signal-to-noise ratio in the PLL circuit and gives the information about the preciseness of phase estimation of incoming signal. It can be assumed \( \rho_{PLL}=1/\sigma_\epsilon^2 \), where \( \sigma_\epsilon \) is a standard deviation of the phase error [7].

The expression for the conditional BER for QPSK, as a function of instantaneous symbol SNR in the channel \( \rho = r^2/2\sigma_n^2 \), \( \sigma_n^2 = n^2(t) \) and phase error \( \epsilon \), can be presented as
\[
P_{b}(\epsilon, \rho)_{QPSK} = \frac{1}{4} \text{erfc} \left( \frac{\rho}{\log_2 M} \left( \cos \epsilon - \sin \epsilon \right) \right) + \frac{1}{4} \text{erfc} \left( \frac{\rho}{\log_2 M} \left( \cos \epsilon + \sin \epsilon \right) \right)
\]
where \( \text{erfc}() \) is the complementary error function [8, Eqn. (8.250)].

The average BER can be obtained by averaging (8) over all possible values of instantaneous symbol SNR \( \rho \) and phase error \( \epsilon \)
\[
P_{b,QPSK} = \int_{-\pi}^{\pi} \int_{0}^{\rho_{max}} P_{b}(\epsilon, \rho)_{QPSK} \rho_{b}(\rho) \rho_{\epsilon}(\epsilon) d\rho d\epsilon,
\]
\[-\pi \leq \epsilon \leq \pi.
\]

III. NUMERICAL RESULTS

Using (6)-(9), one can calculate the average BER for \( K_G \) fading channel and discuss performances of the receiver for different values of \( m_m \) and \( m_s \) parameters, standard
deviation of phase noise, $\sigma_\phi$, as well as for different number of diversity branches, $L$.

In Fig. 1 the influence of shadowing intensity (ms parameter) on BER of QPSK signal detection is presented for different values of fast fading parameter $m_w$. A selection combiner with two branches is used at the reception. Diversity branches are assumed non-identically distributed and fast fading parameters differ in each branch (in this case $m_l$ and $m_w$). A phase error standard deviation is $\sigma_\phi$. One can notice, regardless of the values of $m_l$ and $m_w$ parameters, a BER floor appears when it is $\sigma_\phi = 10^5$. Therefore, no increase of $\rho_{\text{bias}}$ can cause the BER to fall under the certain value. For smaller values of $m_u$ (deeper fast fading) BER floor earlier arises, i.e. for larger average bit BER values. Also, the influence of shadowing on BER is less emphasized in the case of larger fast fading intensity (small $m_u$).

In Fig. 2 shows the influence of phase error on BER, for different values of fast fading parameter $m_u$. For the purpose of comparison, an ideal case, the one without phase error, is given, also. Obviously, the phase error extremely impairs system performance. The phase error of $\sigma_\phi = 10^5$ already brings the BER floor and the increase of $\rho_{\text{bias}}$ can not further improve quality of reception. Of course, with the increase of $\sigma_\phi$, this BER floor appears for lower $\rho_{\text{bias}}$ values. It can be concluded that the quality of PLL circuit in the receiver has a crucial importance.

Fig. 3. Influence of non-identical fading distribution in diversity branches on system performance.

In Fig. 3 one can observe the impact of non-identical fading distribution in diversity branches on system performance. The case of dual branch diversity reception and the different values of fast fading parameters in the first branch, $m_u$, is presented, while fading parameter of the second branch, $m_w$, deviates by 10% and 20% of $m_u$ value. It can be seen that this effect of non-identically distributed branches achieves the greatest impact on the BER in the case of higher fast fading severity (smaller $m_u$).

Fig. 4. Influence of diversity order on BER performance.

The influence of diversity order on the performances of the receiver can be observed from Fig. 4 where dependence of the average BER on average SNR ($\rho_{\text{bias}}$) is shown for different values of parameter $L$. With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity. Power gain is the highest when order of diversity system changes from $L = 1$ to $L = 2$. For example, in order to obtain the same value of BER=$10^{-4}$, for parameter values $m_u = m_w$, $m = 3$ and $\sigma_\phi = 10^5$, it is necessary for average SNR to reach the value of
\( \rho_{01} = 23.7 \text{dB for } L=1, \rho_{02} = 16.44 \text{dB for } L=2, \rho_{03} = 13.98 \text{dB for } L=3, \rho_{04} = 12.74 \text{dB for } L=4, \rho_{05} = 11.8 \text{dB for } L=5, \) and \( \rho_{06} = 11.2 \text{dB for } L=6. \) It can be noticed that the gain exponentially declines with the increase of the order of diversity system.

Fig. 5 shows the influence of fading and shadowing parameters on minimum input average SNR (sensitivity) of the SC receiver required to produce a BER value of \( 10^{-4} \). Branches of the receiver are assumed identically distributed. Curves corresponding to sensitivity of 15dB, 19dB and 22dB are presented for different values of phase error standard deviation \( \sigma_p \). The sensitivity increases with the decrease of \( \sigma_p \) value. This effect is especially emphasized when shadowing and fading severity are low. For large \( \sigma_p \) values the BER floor appears and it totally determines the sensitivity.

![Fig. 5. Required values of average SNR in order to achieve error probability \( 10^{-4} \) for different values of phase error standard deviation \( \sigma_p \).](image)

**IV. CONCLUSION**

In this paper we discussed the detection of QPSK signals in a composite fading channel. The selection combining is applied at the reception, while the branches of the combiner are not necessarily identically distributed. The imperfect carrier signal recovery is taken into account through the phase error that occurs in PLL circuit. It is a random process and follows the Tikhonov distribution. The influence of the fading parameters, the quality of the phase loop circuit and the number of diversity branches on the system performance was examined.

As it was expected, BER decreases when shadowing and fading severity become low. The influence of shadowing parameter becomes determinative when the fast fading severity is low.

The phase error extremely impairs system performance. The phase error of \( \sigma_p = 10^\circ \) already brings the BER floor and the increase of \( \rho_{01} \) can not further improve quality of reception. Of course, with the increase of \( \sigma_p \), this BER floor appears for lower \( \rho_{01} \) values.

The non-identically distributed branches of SC receiver impair the system performance. This effect achieves the greatest impact on the BER in the case of higher fast fading severity.

With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity.

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**REFERENCES**


