Optimal Joint Routing and Power Allocation for Decode-and-Forward Multi-Hop Networks

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Abstract--- A n**ovel cross-layer routing protocol is presented for Decode-and-Forward multi-hop relaying systems which lead to optimal power allocation for a given power budget by maximizing the capacity of source to destination. The problem of rate maximization by jointly allocating power and selecting relays nodes with limited total power constraint of network nodes is formulated when the channel state information (CSI) for all of the links is available and constant during the transmission. Showing the convexity of the problem we had separated the problem into two independent sub-problems without loss of optimality. The first problem related to power allocation and the second problem leads to standard routing problem. Our routing problem has introduced new routing metric which by using it we can hold optimality when applying power allocation to the selected route. Simulation results show that our power aware routing works well in middle SNR too. A comprehensive simulation setup is performed to analyze our algorithm in a realistic scenario which shows large performance gain over the previous power allocation (such as uniform power allocation) and other routing algorithms. Even significant gain is achieved when optimal power allocation or optimal routing algorithm are bundled with non-optimal version of each other.**

Keywords--- decode-and-forward, power allocation, convext optimization, routing, multi-hop relaying

I. INTRODUCTION

Multi-hop relaying systems have become viable approaches to improve the reliability in wireless networks. The widespread wireless applications have an increasing demand for devices with lifetime limited batteries. In some recent studies, optimal power allocation schemes were considered with different criteria and assumptions [1-8]. Some of them focused on two or constant hop relay networks which would not include the routing problem [1-2], [5-6] Optimal power allocation schemes for dual-hop decode-and-forward (DF) system to maximize the instantaneous maximum mutual information, were presented in [1], for amplify-and-forward (AF) relaying in [2]. In [3] for a DF multi-hop transmission system an optimal power allocation scheme is given to minimize the total power consumption subject to achieving a target end-to-end bit error rate. An optimal power allocation scheme that minimized the total power consumption in an amplify-and-forward (AF) multi-hop multi-branch transmission system is proposed in [4]

In [5], optimal power allocation schemes are given for dual-hop employing either DF or AF relaying, and a multi-hop DF relaying system and is aimed at minimizing the outage probability given a certain power. Optimal power allocation schemes that maximize the instantaneous received signal-to-noise ratio (SNR) in an AF multi-hop system for short-term (ST) and long-term (LT) power constraints are given in [6]. In [7] the joint routing and power allocation is presented with the objective function of outage probability from source to destination. The Hop-by-Hop routing strategy to minimize the outage probability is studied in [8]. Though some paper has addressed the power allocation in DF relay system, none of them has investigated the routing and power allocation jointly in the decode-and-forward relaying system in order to maximize the capacity between the source and destination.

In this paper, we present an optimal cross-layer optimization by addressing power aware routing and power allocation scheme separately which maximizes the total capacity of the selected route in a multi-hop single branch DF transmission system subject to a finite power sum.

The link between any two adjacent nodes is assumed to be interference free from other links or in other words, orthogonal to other links while the available bandwidth is equal for all of the links. Separating the joint optimization problems into to two part reduce the complexity of cross-layer optimization problem. The first one is to choose the optimal route to achieve our goal and the second one is to allocate the finite power budget among the relay nodes of the optimal route. Formulating the problem as a convex optimization problem, we achieved a cross-layer routing metric for the DF system which depend only on each link between a couple of nodes thus the routing problem become a dynamic programming problem that can be solved by Dijstera's shortest path algorithm.

The remainder of this paper is organized as follows. In Section II we describe the system model and the problem formulation. Joint optimal routing and power allocation scheme and the separation process is presented in section III. Numerical results are presented in Section IV and section V concludes the paper.

II. SYSTEM MODEL AND THE PROBLEM FORMULATION

We considered a cooperative system with a single branch as depicted in Fig. 1. This branch is composed of $K-1$ relays $R_1, R_2, ... R_{K-1}$. We assume that each node is equipped with only one antenna. Moreover, half-duplex constraints are imposed on the relay nodes (i.e. the relay nodes cannot

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transmit and receive simultaneously). The channels between all the nodes are assumed to be random, independent, frequency-flat and constant over the transmitted block of data. The channel between the relays R_{k-1} and R_k of the branch is denoted by α_k, which is assumed to be a zero mean, circularly symmetric complex Gaussian random variable (r.v) with variance $\sigma_{k,l}^2$. The noise r.v on all links is assumed to be zero mean, independent, additives and Gaussian distributed. Since each links transmission's is orthogonal to the other links, there is no interference in the system. This assumption can be realized by assuming time division multiplexing between all of the network nodes.

Considering DF relaying strategy, each relay decodes the signal received from the immediate preceding node and after encoding forwards to the next node in the next time slot. Although exploiting all the signals transmitted by previous hops can greatly improve the power efficiency, here we assume that a specific receiving node only use signal transmitted by its neighboring relay cluster. The encoding and decoding strategy at the k^{th} relay is adopted based on the instantaneous channel loss or amplitude over the channel between the terminals R_{k-1} and R_k, α_k, to result in a power P_k at the relay.

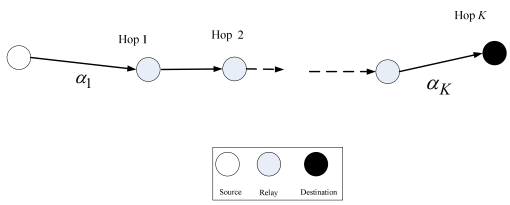

Figure1. Network model with K hops

Where the source transmitter power is P_0 and N_{0_k} is the power of additive white Gaussian noise (AWGN) at the k^{th} relay terminal. It is assumed that the total available power for the system is P_T. In a single-branch multi-hop system with uniform power allocation scheme, equal portions of the total power P_T are assigned to each transmitting node, i.e. $P = \frac{P_T}{K}$. In order to optimize power allocation in this paper we consider that the allocated power to the k^{th} relay terminal is P_{k-1}

So, we can explain all nodes power portions as

$$\sum_{k=1}^{K} P_{k-1} \le P_T \qquad (0)$$

Assuming the loss of direct link between source and destination, maximum capacity through a predefined route from the source to destination can be acquired as

$$C_{DF}^{\ell} = Min\left(\left\{C_{DF,l}\right\}_{l\in\ell}\right) \qquad (0)$$

Where ℓ £ is a set of the link between source and destination and each link is an ordered paired $\left(n_l, n_l'\right)$ of two neighbor nodes related to the l-th link. Also $C_{DF,l}$ is the capacity of l-th link, which can be computed as follow versus the available bandwidth, power and path-loss

$$C_{DF,k} = W \log\left(1 + \frac{p_k \alpha_k}{N_0}\right) \qquad (1)$$

III. JOINT OPTIMAL ROUTING AND POWER ALLOCATION SCHEME

Defining the system model now the joint routing and power allocation problem can be defined as finding the optimal route and power vector which maximize as follow

$$C_{DF} = \underset{\ell,\{\beta_1,..\beta_k\}}{Max}\left(C_{DF}^{\ell}\right) \qquad (1)$$

A. Sepration of Routing and Power Allocation Problems

Now by separating the above Max for the two variables, it can be rewritten as follow

$$C_{DF} = \underset{\ell}{Max}\left(\underset{\{P_1,..P_k\}}{Max}\left(C_{DF}^{\ell}\right)\right) \qquad (1)$$

which separate the two variables $\{P_1,..P_k\}$ and ℓ in two separate maximization problem. It is important to note that the solution of the inner problem (maximization over $\{P_1,..P_k\}$) depend on ℓ, hence the inner maximization have to be solved parametrically for any route ℓ, so that the analytical solution can be used for the outer maximization over ℓ.

Here we denote the inner maximization as power allocation problem and the outer maximization as routing problem.

B. Power Allocation Problem

The power allocation problem can be written as follow

$$\begin{cases} \underset{\{P_1,..P_k\}}{Maximize} \quad t \\ subject\ to \qquad t \le C_{DF,k} = W \log\left(1 + \frac{P_k \alpha_k}{N_0}\right) \quad k = 1...K \\ \qquad\qquad \sum_{k=1}^{K} P_{k-1} \le P_T \end{cases} \qquad (1)$$

It is easy to check that the constraints are convex and the objective function is linear which can be said a concave function. Thus the above maximization problem treated as a standard convex optimization problem. We will rewrite (6) as bellow for some simplifications in verifying the problem conditions.

$$\begin{cases} C_{DF} = \underset{\{P_1,\ldots,P_k\}}{Max}\left(t\right) \\ subject\,to \qquad t'_k \le 0 \quad k = 1\ldots K \\ \qquad\qquad\qquad \sum_{k=1}^{K} P_{k-1} \le P_T \end{cases} \tag{1}$$

Where

$$t'_k = t - C_{DF,k} \tag{2}$$

As it is clear the constraints are differentiable with respect to $P_1,\ldots,P_k$, and it could easily be checked that the Complimentary Slackness condition holds [10] so the inequalities in the conditions hold with equalities. So we can write (7) as bellow

$$\begin{cases} C_{DF} = \underset{\beta_1,\ldots\beta_k}{Max}\left(t\right) \\ subject\,to \qquad t'_k = 0 \quad k = 1\ldots K \\ \qquad\qquad\qquad \sum_{k=1}^{K} P_{k-1} \le P_T \end{cases} \tag{2}$$

For some eases we define the bellow function

$$\gamma_k \triangleq \frac{\alpha_k}{N_0} \tag{3}$$

Then we can write

$$C_{DF,k}(p_k) = W\log\left(1 + p_k\gamma_k\right) \tag{3}$$

So by use of (8), (9) and (11) we have

$$p_k = \frac{\exp(\frac{t}{W}) - 1}{\gamma_k} \tag{3}$$

So we can write (1) as

$$\sum_{k=1}^{K}\left(\frac{\exp(\frac{t}{W}) - 1}{\gamma_k}\right) = P_T \tag{4}$$

then we have

$$z\sum_{k=1}^{K}(\gamma_k^{-1}) = P_T \tag{5}$$

where

$$z = \exp(\frac{t}{W}) - 1 \tag{6}$$

So equation (14) implies that

$$z = P_T \,/ \sum_{k=1}^{K}(\gamma_k^{-1}) \tag{7}$$

Now by use of (12) and (16) the optimal power allocation has been achieved and we have

$$p_k = \frac{P_T}{\gamma_k \sum_{k=1}^{K}(\gamma_k^{-1})} \tag{8}$$

C. Routing Problem

If we consider (8), (9) and (17) we can write

$$C_{DF} = W\log\left(1 + \frac{P_T}{\sum_{k=1}^{K}(\gamma_k^{-1})}\right) \tag{9}$$

As it is clear in above equation for a given power budget to achieve the maximum capacity, we should choose the relays in a manner that maximizes the summation $\sum_{k=1}^{K}(\gamma_k^{-1})$ in denominator; and it would be a dynamic programming problem.

Equation (18) implies to choose γ_k^{-1} as the routing metric to solve this problem, so the problem could be solve by some algorithms like Dijkstra shortest path when the link cost are γ_k^{-1} s. In the next section there are some numerical results to evaluate the optimality of this new scheme.

IV. Numerical Results

To evaluate our new scheme we consider two scenarios to compare it with the common schemes. We consider a rectangle with side length of 60 unit of length as the environment of the example and there are 'n' nods distributed on it randomly with a uniform distribution.

The source and the destination are located on the opposite corners of rectangle with side length of 20 units of length. So there would be a large choice for the routes. In the first scenario we want to just compare our routing scheme compare with Hop-Count routing, to do this, first we should choose a measure of connectivity for the nodes of the network Hop-Count routing. There are two measures considered in Hop-Count routing, the distance and the received SNR, we compare our optimal scheme with the both metrics and to have a fair comparison we run some precise simulations to choose optimal metrics for Hop-Count routing not just choose some heuristic one. Figure1 shows the optimal distance metric versus the number of nodes

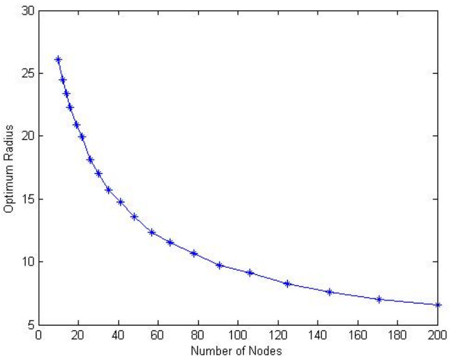

Figure 2 Optimal Connectivity Radius for Hop-Count Algorithm

As it was expected the optimal radius has been decreased as the number of nodes increases and either for the small number of nodes the optimal radius is near the distance between the source and destination, which is the diameter of the rectangular. Figure2 shows the optimal received SNR metric versus the number of nodes.

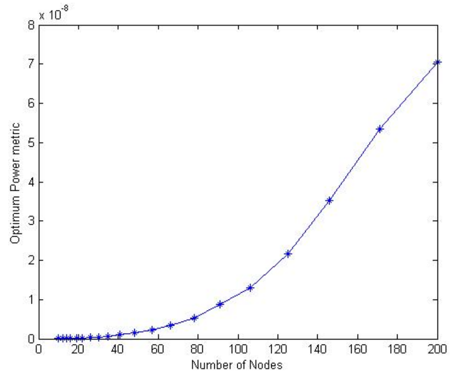

Figure 3 Optimal Connectivity SNR measure for Hop-Count Algorithm

The optimal received SNR metric has been increased as the number of nodes increased, and it was expected, because when we have more neighbors we could choose one who received the message with higher SNR. After this, in our simulations for any number of nodes, we choose the optimum metrics for the Hop-Count routing from these results to be compared with our scheme. For the following simulations we distribute a constant number of nodes in each iteration, 106 nodes, and the results are the ensemble average of more than 1000 iterations.

Because of brevity we will discuss just on the numerical results which could be mentioned to evaluate the new scheme. In this scenario we want to apply the optimal power allocation to our new scheme and compare its performance compare with the Hop-Counts algorithms with uniform power allocation. The result is depicted in Figure 4.

As it is clear in Figure6 the new scheme obtains more than 35 dB performance gain compare with the Hop-Count scheme with SNR metric and distance metric when the optimum power allocation is applied.

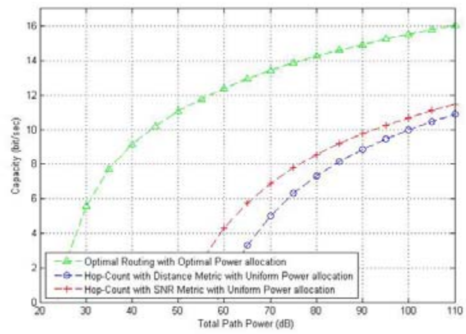

Figure 4 Capacity versus Power Budget with Optimal Power Allocation

V. CONCLUTION

In this paper we described the problem of joint maximization of rate, relay selection and optimal power allocation while having a limited total power constraint on the relay network. Formulating the optimal power allocation algorithm in this paper we deduce the routing metric for maximization of the capacity. Then the routing procedure which was a dynamic programming problem was solved by the conventional algorithms like Dijkstra shortest path algorithm by choosing the new metric as the link cost in the network.

VI. ACKNOWLEDGMENT

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