Moments of the signal after micro- and macrodiversity processing in gamma shadowed Nakagami-*m* fading channels

Nikola M. Sekulović, Edis S. Mekić, Mihajlo Č. Stefanović, Aleksandra D. Cvetković, and Selena Ž. Stanojčić

***Abstract* — This paper studies wireless communication system with micro- and macrodiversity reception in gamma shadowed Nakagami-*m* fading channels. *N*-branch maximal-ratio combining (MRC) is implemented at the micro level (single base station) and selection combining (SC) with two base stations (dual diversity) is implemented at the macro level. Expression for the moments, which is very useful since it can be used to directly obtain important system performance measures, such as the average signal value and amount of fading (AoF), is analytically derived. Various numerical results are graphically presented to illustrate the proposed mathematical analysis and to show the effects of various system parameters to the system performance, as well as enhancement due to use of the combination of micro- and macrodiversity.**

Keywords — Gamma shadowing, macrodiversity, microdiversity, Nakagami-*m* fading.

I. Introduction

IN wireless communication systems, the received signal can be affected by both short-term fading and long-term fading (shadowing). Short-term fading is the result of multipath propagation while shadowing is the result of large obstacles between transmitter and receiver [1]. Diversity combining, which combines multiple replicas of the received signal, is a very efficient technique to mitigate these detrimental effects and to improve the performance of wireless communications systems at relatively low cost [2]. Space diversity [3], [4], achieved by using multiple receive antennas, is the most common form of diversity. The most popular space diversity techniques are selection combining (SC), equal-gain combining (EGC) and maximal-ratio combining (MRC).

Diversity techniques at single base station (microdiversity) reduce the effects of short-term fading. Impairments due to shadowing can be mitigated using macrodiversity techniques which employ the processing of signals from multiple base stations. The use of composite micro- and macrodiversity has received considerable interest due to the fact that it simultaneously combats both short-term fading and shadowing.

Fading envelope of the received signal is usually described with Rayleigh, Rician, Nakagami-*m* and Weibull statistical models. The average power, which is random variable due to shadowing, is usually modeled with lognormal distribution. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal is considered in [5]-[8]. Unfortunately, the use of lognormal distribution to account for shadowing does not lead to a closed-form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) after micro- and macrodiversity combining making the analysis of system in shadowed fading environment very ponderous. Based on theoretical results and measured data, it was shown that gamma distribution does the job as well as lognormal [9], [10]. A compound fading model incorporates both short-term fading and shadowing which is modeled using gamma distribution instead of lognormal distribution [10]-[15].

In this paper, we investigate wireless communication system with micro- and macrodiversity reception using respectively MRC and SC in a correlated gamma shadowed Nakagami-*m* fading channels. Analytical expression for the moments of signal after processing at the micro and macro level is derived. The analytical expression for the moments is very useful since it can be used to directly obtain important performance measures, such as the average signal value and amount of fading (AoF). Numerical results are graphically presented to show the effects of fading severity, shadowing severity, number of diversity branches at the micro level and correlation coefficient on system performance, as well as enhancement due to use of the combination of micro- and macrodiversity.

II. System model

Macrodiversity composed of two geographically distributed microdiversity systems (base stations) per cell operating over gamma shadowed Nakagami-*m* fading

This work was supported in part by the Ministry of Science of Serbia within the project "Development and realization of new generation software, hardware and services based on software radio for specific purpose applications" (TR-11030).

Nikola M. Sekulović, Edis S. Mekić, Mihajlo Č. Stefanović, Aleksandra D. Cvetković, Selena Ž. Stanojčić are with the Faculty of Electronic Engineering, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia.

Corresponding author Nikola M. Sekulović (phone: +381-62-8281360, e-mail: sekulani@gmail.com).

channels is analyzed in this paper (see Fig. 1). N-branch MRC receiver is implemented at the micro level (single base station) and SC receiver which involves the use of two geographically distributed base stations (dual diversity) is implemented at the macro level. The macrodiversity SC scheme is based on the simple selection of the base station with the larger average power. The improvement in performance gained through the use of multiple antennas at the single base station is larger when the signals corresponding to each antenna are approximately independent, i.e. when the separation between antennas is on the order of one half of a wavelength. Two base stations are treated to have nonzero correlation. It is realistic scenario because shadowing has a larger correlation distance and it is difficult to ensure that base stations operate independently, especially in microcellular systems [6].

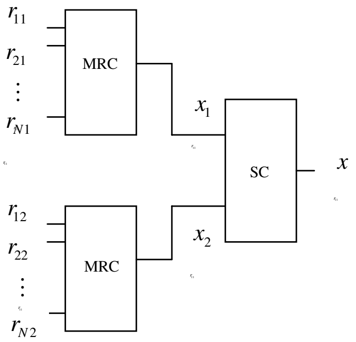

Fig. 1 System model

The PDF of the signal received by the ith antenna at the jth base station in the presence of Nakagami-m fading is

$$p_{r_{ij}}(r_{ij})=\frac{2m^m r_{ij}^{2m-1}}{\Gamma(m)\Omega_j^{\,m}}\exp\left(-\frac{m}{\Omega_j}r_{ij}^2\right)\quad,\quad i=\overline{1,N}\ ,\ j=1,2 \qquad (1)$$

where $\Gamma(\cdot)$ is gamma function, Ω_j is the average power of the signal at the jth base station and m is Nakagami fading parameter which describes fading severity ($m\geq 0.5$). As parameter m increases, the fading severity decreases. After transformation $x_{ij}=r_{ij}^2$, (1) becomes

$$p_{x_{ij}}(x_{ij})=\frac{m^m x_{ij}^{m-1}}{\Gamma(m)\Omega_j^{\,m}}\exp\left(-\frac{m}{\Omega_j}x_{ij}\right)\quad,\quad i=\overline{1,N}\ ,\ j=1,2\ . \qquad (2)$$

The result signal at the MRC output of the jth base station is the sum of squared envelopes of Nakagami-m faded signals, $x_j=\sum_{i=1}^{N} r_{ij}^2$, or equivalently, $x_j=\sum_{i=1}^{N} x_{ij}$ with PDF given by [16, (71)]

$$p_{x_j}(x_j\,|\,y_j)=\frac{x_j^{\,M-1}M^M}{\Gamma(M)\,y_j^{\,M}}\exp\left(-\frac{M}{y_j}x_j\right),\quad j=1,2 \qquad (3)$$

where y_j is the total input power ($y_j=N\Omega_j$) and $M=Nm$.

The conditional nature of the PDF in (3) reflects the existence of shadowing with y_j being random variable. The joint PDF of y_1 and y_2 follows the correlated gamma distribution [17], [18]

$$p_{y_1y_2}(y_1,y_2)=\frac{\rho^{-\frac{c-1}{2}}(y_1y_2)^{\frac{c-1}{2}}}{\Gamma(c)(1-\rho)\,y_0^{c+1}}\exp\left(-\frac{y_1+y_2}{(1-\rho)\,y_0}\right) \times I_{c-1}\left(\frac{2\sqrt{\rho y_1y_2}}{(1-\rho)\,y_0}\right) \qquad (4)$$

where ρ is the correlation between y_1 and y_2, c is the order of gamma distribution, y_0 is related to the average power of y_1 and y_2, $I_n(\cdot)$ is the first kind and nth order modified Bessel function. The severity of gamma shadowing is measured in terms of c. The lower value of c means the higher shadowing while the value of $c=\infty$ corresponds to a pure short-term fading channel. The relationship between the parameter c and standard deviation σ of shadowing in dB in the lognormal shadowing exists through $\sigma(dB)=4.3429\sqrt{\psi'(c)}$, where $\psi'(\cdot)$ is the trigamma function. The typical values of σ are between 2-12 dB.

Selection diversity is applied at the macrolevel. Namely, the base station with the larger average power is selected to provide service to the user. Using the concepts of probability, the PDF of x after diversity combining at the micro- and macrolevel can be derived as

$$\begin{aligned}p_x(x)&=\int_0^\infty dy_1\int_0^{y_1}p_{x_1}(x\,|\,y_1)\,p_{y_1y_2}(y_1,y_2)\ dy_2\\&+\int_0^\infty dy_2\int_0^{y_2}p_{x_2}(x\,|\,y_2)\,p_{y_1y_2}(y_1,y_2)\ dy_1\\&=2\int_0^\infty p_{x_1}(x\,|\,y_1)\,dy_1\int_0^{y_1}p_{y_1y_2}(y_1,y_2)\ dy_2\,.\end{aligned} \qquad (5)$$

which, by substituting (3) and (4) and using [19, eqs. (8.445), (3.381/2) and (3.471/9)], yields

$$\begin{aligned}p_x(x)&=\frac{1}{\Gamma(M)\Gamma(c)}\sum_{n,k=0}^{\infty}\frac{\rho^n M^{n+c+\frac{k+M}{2}}}{2^{n+c+\frac{k-M}{2}-2}(1-\rho)^{n+\frac{k+M}{2}}}\\&\times\frac{x^{n+c+\frac{k+M}{2}-1}}{n!\,\Gamma(n+c)\,y_0^{n+c+\frac{k+M}{2}}\prod_{l=0}^{k}(n+c+l)}\\&\times K_{2(n+c)+k-M}\left(2\sqrt{\frac{2Mx}{y_0(1-\rho)}}\right).\end{aligned} \qquad (6)$$

III. MOMENTS OF THE RECEIVED SIGNAL

The Lth order moment of the received signal can be derived as [20, eq. (5-38)]

$$x_L=\varepsilon\langle x^L\rangle=\int_0^\infty x^L p_x(x)\,dx\ . \qquad (7)$$

Substituting (6) in (7) and using [19, eq. (6.561/16)], Lth moment can be written as

$$\begin{aligned}x_L&=\frac{1}{\Gamma(M)\Gamma(c)}\\&\times\sum_{n,k=0}^{\infty}\frac{\rho^n(1-\rho)^{L+c}\,y_0^{\,L}\,\Gamma(L+2(n+c)+k)\,\Gamma(L+M)}{2^{L+2(n+c)+k-1}\,n!\,M^L\,\Gamma(n+c)\prod_{l=0}^{k}(n+l+c)}.\end{aligned} \qquad (8)$$

A composite multipath/shadowed fading environment can be described by using the average signal value and AoF. The analytical expression for the moments is very useful since it can be used to directly obtain these important performance measures. The average signal value can be obtained by setting L=1 in (8), i.e.

$$\bar{x} = x_1. \quad (9)$$

The AoF is can be expressed in terms of first and second order moments as

$$AoF = \frac{x_2}{x_1^2} - 1 \;. \quad (10)$$

Typically, this performance measure is independent of the average power. The higher order moments (L>2) are also useful in signal processing algorithms for signal detection, classification, and estimation and they play a fundamental role in studying the performance of wideband communication systems.

IV. Numerical results

Using the previous mathematical analysis, performance evaluation results for the proposed communication system are presented in this section.

Table 1: The number of terms needed to be summed in (9) to achieve accuracy at the 4th significant digit (M=2.2)

y_0(dB)	ρ=0.2			ρ=0.6		
	c=0.3	c=1.2	c=5.2	c=0.3	c=1.2	c=5.2
0	14	17	25	24	23	41
5	15	22	29	25	27	42
10	18	22	30	25	29	44

Table II The number of terms needed to be summed in (10) to achieve accuracy at the 4th significant digit

c	ρ=0.2		ρ=0.4		ρ=0.6	
	M=2.2	M=4.4	M=2.2	M=4.4	M=2.2	M=4.4
0.2	21	20	23	24	27	26
3	21	20	23	24	30	31
6	21	19	23	23	28	32

Analytical expressions for the average signal value and AoF are in the form of the nested infinite multiple sums. But, these expressions converge rapidly, and therefore, they can be efficiently used. Tables I and II summarize the number of terms needed to be summed in expressions for the average signal value and AoF, respectively, to achieve accuracy at the 4th significant digit.

In Fig. 2, the average signal value is plotted as a function of y_0. As it was expected, system performance improves with an decrease of correlation coefficient and shadowing severity. In this figure, we also wanted to see and establish improvement obtained through macrodiversity. For example, for M=2.2, at an $\bar{x}$ of 25, the macrodiversity gain for c=5.2 and ρ=0.2 is about 0.9 dB, for ρ=0.6, 0.7 dB; for c=0.3 and ρ=0.2 is about 2.2 dB while for ρ=0.6 is 1.6 dB. The macrodiversity gain here is defined as the reduction in the y_0 compared with the case having no macrodiversity (see Appendix). Therefore, it can be deduced clear conclusion that the combination of micro- and macrodiversity provides significant performance improvement, especially in environment under higher shadowing severity.

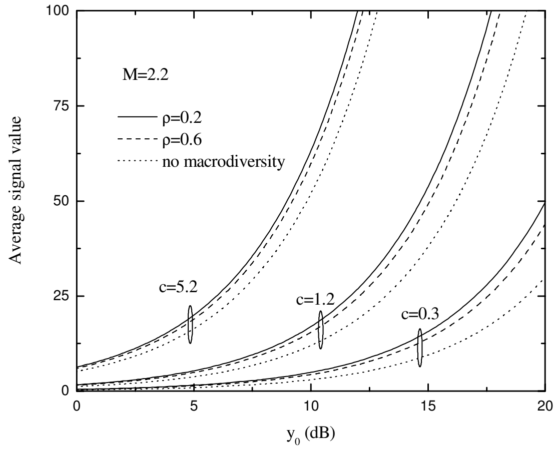

Figure 2. Average signal value versus y_0 for different correlation coefficient and shadowing severity

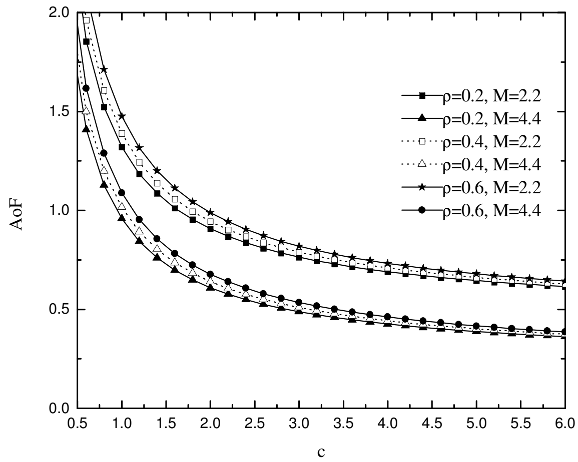

Figure 3. AoF versus c for different values of parameter M and correlation coefficient

Fig. 3 presents the AoF versus c for different values of parameter M and correlation coefficient. It is observed that AoF decreases with an increase of c and/or M, while when ρ increases, AoF also increases resulting in performance degradation.

V. Conclusion

In this paper, system with micro- and macrodiversity reception was considered. The received signal envelope has a Nakagami-m distribution and it also suffers gamma shadowing. Microdiversity scheme is based on N-branch MRC and macrodiversity scheme is based on dual SC. Expression for the moments was analytically derived. This expression is very useful since it can be used to directly obtain important system performance measures, such as the average signal value and AoF. They require the summation of an infinite number of terms. However, the presented infinite-series representations converge for any value of the parameters and accordingly, they enable great accuracy of the evaluated and graphically presented results. Numerical results show that the system performance improves with an increase of the shadowing parameter c and/or parameter M while an increase of the correlation coefficient leads to deterioration of the system performance. Improvement achieved through macrodiversity is also established.

APPENDIX: The case of no macrodiversity

The PDF of the signal at the output of single base station (the case of no macrodiversity) in shadowed Nakagami-*m* fading channels is

$$p_x(x) = \int_0^\infty p_x(x|y)\, p_y(y)\, dy \tag{11}$$

where $p_y(y)$ is the gamma PDF of the average power given by [18]

$$p_y(y) = \frac{y^{c-1}\exp(-y/y_0)}{\Gamma(c)\, y_0{}^c}. \tag{12}$$

Substituting (3) and (12) in (11) and after some straightforward manipulations, integral can be solved with the use of [19, (3.471/9)], resulting in analytical expression for the PDF of x

$$p_x(x) = \frac{2M^{\frac{c+M}{2}} x^{\frac{c+M}{2}-1}}{\Gamma(M)\Gamma(c)\, y_0^{\frac{c+M}{2}}} K_{c-M}\left(2\sqrt{\frac{Mx}{y_0}}\right). \tag{13}$$

Substituting (13) in (7) and using [19, (6.561/16)], *L*th moment for the case with no macrodiversity can be written as

$$x_L = \frac{y_0{}^L \Gamma(c+L)\Gamma(M+L)}{\Gamma(M)\Gamma(c)M^L}. \tag{14}$$

REFERENCES

[1] D. Tse and P. Viswanath, "Fundamentals of Wireless Communication", Cambridge University Press, July 2005.

[2] E. Biglieri, "Coding for Wireless Channels", New York: Springer, 2005.

[3] M. K. Simon and M.-S. Alouini, "Digital Communication Over Fading Channels", 1st ed. New York: Wiley, July 2000

[4] A. Goldsmith, "Wireless Communications", Cambridge University Press, Aug. 2005.

[5] F. Hansen and F. I. Mano, "Mobile fading-Rayleigh and lognormal superimposed," *IEEE Trans. Veh. Tech.,* vol. 26, no. 4, pp. 332–335, Nov. 1977.

[6] J. Zhang and V. Aalo, "Effect of macrodiversity on average-error probabilities in a Rician fading channel with correlated lognormal shadowing," *IEEE Trans. Commun.,* vol. 49, no. 1, pp. 14-18, Jan. 2001.

[7] E. K. Al-Hussaini, A. M. Al-Bassiouni, H. Mouradand, and H. Al-Shennawy, "Composite macroscopic and microscopic diversity of sectorized macrocellular and microcellular mobile radio systems employing RAKE receiver over Nakagami fading plus lognormal shadowing channel," *Wireless Personal Commun.*, vol. 21, pp. 309–328, June 2002.

[8] A. Panajotovic, M. Stefanovic, and D. Draca, "Effect of microdiversity and macrodiversity on average bit error probability in shadowed fading channels in the presence of interference," *ETRI J.*, vol. 31, no. 5, pp. 500-505, Oct. 2009.

[9] A. Abdi and M. Kaveh, "On the utility of gamma PDF in modeling shadow fading (slow fading)," in Proc. IEEE Veh. Tech. Conf., Houston*, TX*, vol. 3, pp. 2308-2312, July 1999.

[10] P. M. Shankar, "Performance analysis of diversity combining algorithms in shadowed fading channels," *Wireless Personal Commun.*, vol. 37, pp. 61-72, Apr. 2006.

[11] I. Kostic, "Analytical approach to performance analysis for channel subject to shadowing and fading," *IEE Proc. Comm.*, vol. 152, no. 6, pp. 821-827, Dec. 2005.

[12] P. M. Shankar, "Analysis of microdiversity and dual channel macrodiversity in shadowed fading channels using a compound fading model," *Int J Electron Comm (AEÜ)*, vol. 62, no. 6, pp. 445-449, June 2008.

[13] V. Milenkovic, N. Sekulovic, M. Stefanovic, and M. Petrovic, "Effect of microdiversity and macrodiversity on average bit error probability in gamma shadowed Rician fading channels,"*ETRI J.*, vol. 32, no. 3, pp. 464-467, June 2010.

[14] N. Sekulović, D. Drača, A. Panajotović, Z. Nikolić and Č. Stefanović, "Channel Capacity of a System in Shadowed fading Channels with Micro- and Macrodiversity Reception", *XLIV International Scientific Conference on Information, Communication and Energy Systems and Technologies - ICEST 2009*, Veliko Tarnovo, Bulgaria, 25-27 June 2009, *Proceedings of papers*, vol. 1, pp.82-84.

[15] N. Sekulovic, E. Mekic, D. Krstic, I. Temelkovski, D. Manic and M. Stefanovic, "Outage probability of macrodiversity system in Nakagami-m fading channels with correlated gamma shadowing", *Proceedings of the International Conference on Circuits, Systems, Signals*, Malta (2010), pp. 266-270.

[16] M. Nakagami, "The m-distribution - A general formula of intensity distribution of rapid fading," *in Statistical Methods in Radio Wave Propagation*, Pergamon Press, Oxford, U.K., pp. 3-36, 1960.

[17] S. Yue, TBMJ Ouarda, and B. Bobee, "A review of bivariate gamma distributions for hydrological application," *J. Hydrol.*, vol. 246, pp. 1-18, June 2001.

[18] E. Xekalaki, J. Panaretos, and S. Psarakis, "A Predictive Model Evaluation and Selection Approach - The Correlated Gamma Ratio Distribution," STOCHASTIC MUSINGS: PERSPECTIVES FROM THE PIONEERS OF THE LATE 20TH CENTURY, J. Panaretos, ed., Laurence Erlbaum, Publisher, USA, pp. 188-202, 2003. Available: http://ssrn.com/abstract=947067

[19] I. S. Gradshteyn and I. M. Ryzhik, "Table of integrals, series, and products", *Academic*, New York, 5th edn., 1994.

[20] A. Papoulis, "Probability, Random Variables, and Stochastic Processes", 3rd ed., New York: McGraw-Hill, Feb. 1991.