EM-based Signal Detection for Space Time Block Coded MIMO OFDM Systems

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***Abstract*— In this paper, a signal detection technique based on EM algorithm is proposed for space time block coded MIMO-OFDM systems. The observed received data is superposition of all the transmitted signals from all users. This superimposed received signals are decomposed into their signal components and then the signal detection is done by using these signal components. Simulation results show that for two users with increasing the number of EM iteration, the bit error rate equal to zero even when the SNR is low. But for three or more users, the bit error rate decreases until the certain number of EM iteration and then it remains almost constant.**

***Index Terms*— EM, MIMO-OFDM, space time block coded.**

I. Introduction

The goal of future wireless communication is to provide high quality wireless multimedia services, and thus high data rate communication must be transmitted reliably. For higher data rate, more bandwith is required. Orthogonal frequency division multiplexing (OFDM) has high spectrum efficiency. One of the advantages of OFDM systems is to convert the frequency selective channels to several flat subchannels. In OFDM transmission, the problem of intersymbol interference is avoided by the addition of cyclic prefix [1]. Multiple transmit and receive antennas, which are usually referred to MIMO systems, enhances diversity gain by developing a special space time coding and increases the information capacity of wireless communication systems by applying interference cancellation technique [2] [3] [4]. The combination of MIMO and OFDM is a strong condidate for the recent wireless communications systems. Space time block code help increasing reliability over wireless communications [5], and they can achieve full diversity gains with simple linear processing at the receiver. The EM algorithm is used to compute maximum likelihood estimates of system parameters given incomplete observed data. The EM algorithm consists of two iterative steps, expectation and maximization. The EM algorithm first finds the conditional expectation of the joint log-likelihood conditioned upon the incomplete observation. The maximization step then provides new estimates of the parameters that maximize the expectation of the joint log-likelihood defined over complete data conditioned on the most recent observation and last estimate [6]. The V-BLAST detection is reported in [7] that is included linear detection and SIC detection(The detected symbol will be canceled the receive signal and this effects the exactly evaluation for the symbols remain). The joint over antenna (JA) signal detection algorithm based on two minimum mean squared error (MMSE) criteria suppresses interference while preserving the space time coded signal [8]. In this paper, we propose a signal detection technique based on EM algorithm for space time block coded MIMO-OFDM systems. Symbol detection is done by decomposing the superimposed received signals. Simulation results and analysis show that the effect of noise decrease with increasing the number of EM iteration as error of symbol estimation approach to zero. Rate of decreasing of noise effect for two users is very high as error of symbol estimation equal to zero for the few number of EM iteration. Comparing two users, three users have more errors of symbol estimation for the same number of EM iteration. The rest of the paper is organized as follows. In section II, we will describe the MIMO-OFDM system model and discuss some assumptions. Section III explains the EM based algorithm for signal detection of STBC MIMO-OFDM systems. EM-based symbol detection algorithm is developed for two and three users in section IV. Section V provides simulation results of the proposed EM algorithm for two and three users . Finally, we draw some conclusions in Section VI.

II. System Model

Assume we have M users that each of them uses two transmit antennas and NR receive antennas. Each user transmits a space time block coded signal according to the Alamouti scheme [2]. It is assumed that the channels is constant at two consecutive blocks. The cyclic prefix of length ν in order to avoid inter-block interference is added to the each symbol

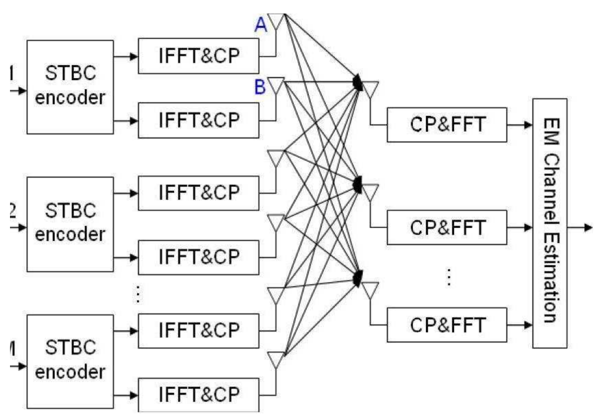

Fig. 1. Block diagram of M user system with 2M transmit and M receive antennas.

data blocks $x^{(i)}_{k,1}$ and $x^{(i)}_{k,2}$, $k = 0, 2, 4, ...$, and then those are transmitted from two transmit antennas. Fig.1 shows the system block diagram. The received blocks k and $k+1$ at the l^{th} antenna can be written as:

$$r_{k,l} = \sum_{i=1}^{M} (h^{(i)}_{1,l} x^{(i)}_{k,1} + h^{(i)}_{2,l} x^{(i)}_{k,2}) + n_{k,l} \quad (1)$$

$$r^*_{k+1,l} = \sum_{i=1}^{M} (-h^{*(i)}_{1,l} x^{(i)}_{k,2} + h^{*(i)}_{2,l} x^{(i)}_{k,1}) + n^*_{k+1,l} \quad (2)$$

where $(.)^*$ denotes complex conjugation, and $h^{(i)}_{m,l}$ is the circulant channel matrix from the m^{th} transmit antenna of the i^{th} user to the l^{th} receive antenna, of the form:

$$h^{(i)} = \begin{bmatrix} h^{(i)}(0) & 0 & \dots & h^{(i)}(\nu) & \dots & h^{(i)}(1) \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h^{(i)}(\nu-1) & \dots & h^{(i)}(0) & 0 & \dots & h^{(i)}(\nu) \\ h^{(i)}(\nu) & h^{(i)}(\nu-1) & \dots & h^{(i)}(0) & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & h^{(i)}(\nu) & h^{(i)}(\nu-1) & \dots & h^{(i)}(0) \end{bmatrix}$$

III. EM-BASED SYMBOL DETECTION ALGORITHM

In this section, we will introduce the EM-based algorithm for symbol detection of STBC MIMO OFDM systems. The idea of EM algorithm is to decompose the observed data into its signal components and then estimate the transmited symbol of each users separately. The algorithm iterates back and forth, using the current symbol estimation to decompose the observed data better and thus improve the next symbol estimation. Applying the DFT matrix D to both sides of (1) and (2), we can obtain the transformed observations:

$$Y_{k,l} = \sum_{i=1}^{M} (\mathbf{\Omega}^{(i)}_{1,l} X^{(i)}_{k,1} + \mathbf{\Omega}^{(i)}_{2,l} X^{(i)}_{k,2}) + N_{k,l} \quad (3)$$

$$Y^*_{k+1,l} = \sum_{i=1}^{M} (-\mathbf{\Omega}^{*(i)}_{1,l} X^{(i)}_{k,2} + \mathbf{\Omega}^{*(i)}_{2,l} X^{(i)}_{k,1}) + N^*_{k+1,l} \quad (4)$$

where $Y = Dr, X = Dx, N = Dn, \mathbf{\Omega}^{(i)}_{1,l} = Dh^{(i)}_{1,l} D^*$ and $\mathbf{\Omega}^{(i)}_{2,l} = Dh^{(i)}_{2,l} D^*$ (diagonalized matrices). Then the m^{th} component of the received vectors at the l^{th} antenna can be written as:

$$\begin{bmatrix} Y_{k,l}(m) \\ Y^*_{k+1,l}(m) \end{bmatrix} = \sum_{i=1}^{M} \begin{bmatrix} \mathbf{\Omega}^{(i)}_{1,l}(m) & \mathbf{\Omega}^{(i)}_{2,l}(m) \\ \mathbf{\Omega}^{*(i)}_{2,l}(m) & -\mathbf{\Omega}^{*(i)}_{1,l}(m) \end{bmatrix} \begin{bmatrix} X^{(i)}_{k,1}(m) \\ X^{(i)}_{k,2}(m) \end{bmatrix} + \begin{bmatrix} N_{k,l}(m) \\ N^*_{k+1,l}(m) \end{bmatrix} \quad (5)$$

Let $\mathbf{Q}_l(m) = \begin{bmatrix} Y_{k,l}(m) \\ Y^*_{k+1,l}(m) \end{bmatrix}$, $\mathbf{S}^{(i)}(m) = \begin{bmatrix} X^{(i)}_{k,1}(m) \\ X^{(i)}_{k,2}(m) \end{bmatrix}$, $\mathbf{\Lambda}^{(i)}_l(m) = \begin{bmatrix} \mathbf{\Omega}^{(i)}_{1,l}(m) & \mathbf{\Omega}^{(i)}_{2,l}(m) \\ \mathbf{\Omega}^{*(i)}_{2,l}(m) & -\mathbf{\Omega}^{*(i)}_{1,l}(m) \end{bmatrix}$, and $\eta_l(m) = \begin{bmatrix} N_{k,l}(m) \\ N^*_{k+1,l}(m) \end{bmatrix}$, then (5) can be written as:

$$\mathbf{Q}_l(m) = \sum_{i=1}^{M} \mathbf{\Lambda}^{(i)}_l(m) \mathbf{S}^{(i)}(m) + \eta_l(m) \quad (6)$$

where $\mathbf{\Lambda}^{(i)}_l(m)$ has an Alamouti-like structure. In this equation $\mathbf{Q}_l(m)$ is the incomplete data and we want to estimate $\mathbf{S}^{(i)}(m)$ for $m = 0, 1, \dots, N-1$ and $i = 1, 2, \dots, M$ by decomposing the incomplete observed data into complete data [6]. The complete data is the component of observed data that is transmitted by each user that is written as:

$$\mathbf{Q}^{(i)}_l(m) = \mathbf{\Lambda}^{(i)}_l(m) \mathbf{S}^{(i)}(m) + \eta^{(i)}_l(m) \quad (7)$$

where $\sum_{i=1}^{M} \mathbf{Q}^{(i)}_l(m) = \mathbf{Q}_l(m)$ and $\sum_{i=1}^{M} \eta^{(i)}_l(m) = \eta_l(m)$. The EM-algorithm can then be expressed as follows [6]:
E-step: For $i = 1, 2, \dots, M$ compute:

$$\mathbf{Z}^{(i)^k}_l(m) = \mathbf{\Lambda}^{(i)}_l(m) \mathbf{S}^{(i)^k}(m) \quad (8)$$

$$\hat{\mathbf{Q}}^{(i)^k}_l(m) = \mathbf{Z}^{(i)^k}_l(m) + \beta^{(i)} \Big(\mathbf{Q}_l(m) - \sum_{j=1}^{M} \mathbf{Z}^{(j)^k}_l(m) \Big) \quad (9)$$

where $\hat{\mathbf{Q}}^{(i)}_l(m)$ is the complete data, and $\sum_{i=1}^{M} \beta^{(i)} = 1, \beta^{(i)} \geq 0$. M-step: For user $i = 1, 2, \dots, M$ compute:

$$min \left\| \hat{\mathbf{Q}}^{(i)^k}_l(m) - \mathbf{\Lambda}^{(i)}_l(m) \mathbf{S}^{(i)^k}(m) \right\|^2 \quad (10)$$

The $(k+1)^{th}$ symbol estimation is obtained as [9]:

$$\hat{\mathbf{S}}^{(i)^{k+1}}(m) = \Big[\mathbf{\Lambda}^{(i)^H}_l(m) \mathbf{\Lambda}^{(i)}_l(m) \Big]^{-1} \mathbf{\Lambda}^{(i)^H}_l(m) \hat{\mathbf{Q}}^{(i)^k}_l(m) \quad (11)$$

where $(.)^H$ denotes the Hermitian.
Initialization: The selection of initial value in the EM algorithm is very important. The initial estimate of $\mathbf{S}^{(i)^0}(m)$ for first EM iteration can be obtained by each user transmitting signal at each two consecutive time units while other users transmit nothing [9]. For $i = 1, 2, \dots, M$ we can write:

$$\mathbf{Q}^{(i)}_l(m) = \mathbf{\Lambda}^{(i)}_l(m) \mathbf{S}^{(i)}(m) + \eta_l(m) \quad (12)$$

$$\hat{\mathbf{S}}^{(i)^0}(m) = \Big[\mathbf{\Lambda}^{(i)^H}_l(m) \mathbf{\Lambda}^{(i)}_l(m) \Big]^{-1} \mathbf{\Lambda}^{(i)^H}_l(m) \mathbf{Q}^{(i)}_l(m) \quad (13)$$

Notice that $\beta^{(i)}$ can be arbitrarily selected, however we can always choose $\beta^{(i)} = \frac{1}{M}$.

IV. ANALYSIS OBTAINED FORMULAS FOR TWO AND THREE USERS

In this section, first we assume that there are two users and then analysis obtained formulas from EM algorithm and then we do that for three users. The $\mathbf{\Lambda}^{(i)}_l(m)$ has an Alamouti-like structure and it is Hermitian matrix then $\Big[\mathbf{\Lambda}^{(i)^H}_l(m) \mathbf{\Lambda}^{(i)}_l(m) \Big]^{-1} = \frac{I}{|\mathbf{\Omega}^{(i)}_{1,l}(m)|^2 + |\mathbf{\Omega}^{(i)}_{2,l}(m)|^2}$, where I is identity matrix. With inserting equation (12) into equation (13) is obtained:

$$\hat{\mathbf{S}}^{(i)^0}(m) = \mathbf{S}^{(i)}(m) + \mathbf{\Gamma}^{(i)}_l(m) \quad (14)$$

where $\mathbf{\Gamma}^{(i)}_l(m) = \frac{\mathbf{\Lambda}^{(i)^H}_l(m) \eta_l(m)}{|\mathbf{\Omega}^{(i)}_{1,l}(m)|^2 + |\mathbf{\Omega}^{(i)}_{2,l}(m)|^2}$.

A. Two users

For the first iteration of EM algorithm, according to equation (14) can be written:

$$\begin{aligned}\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)^0}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)^0}(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)\\ &+2\eta_l(m)\end{aligned}\tag{15}$$

where $\mathbf{\Lambda}_l^{(i)}(m)\mathbf{\Gamma}_l^{(i)}(m)=\eta_l(m)$. According to equation (6) we have:

$$\mathbf{Q}_l(m)=\mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)+\eta_l(m)\tag{16}$$

thus $\beta^{(i)}\Big(\mathbf{Q}_l(m)-\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m)\Big)=-\frac{1}{2}\eta_l(m)$. where $\beta^{(i)}=\frac{1}{2}$ for $i=1,2$. With inserting the above result at the equation (9), is obtained:

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(1)^0}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\eta_l(m)-\frac{1}{2}\eta_l(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\frac{1}{2}\eta_l(m)\end{aligned}\tag{17}$$

With inserting equation (17) at the equation (11), next symbol estimation is calculated as:

$$\begin{aligned}\hat{\mathbf{S}}^{(1)^1}(m) &= \frac{\mathbf{\Lambda}_l^{(1)^H}(m)\hat{\mathbf{Q}}_l^{(1)^0}(m)}{|\mathbf{\Omega}_{1,l}^{(1)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(1)}(m)|^2}\\ &= \mathbf{S}^{(1)}(m)+\frac{1}{2}\mathbf{\Gamma}_l^{(1)}(m).\end{aligned}\tag{18}$$

At the second iteration of EM algorithm, we repeat above steps with using the new symbol estimation as:

$$\begin{aligned}\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^1}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)^1}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)^0}(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)\\ &+\frac{3}{2}\eta_l(m)\end{aligned}\tag{19}$$

The $\mathbf{Q}_l(m)$ is constant and is obtained from equation (16).

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(1)^1}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\frac{1}{2}\eta_l(m)-\frac{1}{4}\eta_l(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\frac{1}{4}\eta_l(m)\end{aligned}\tag{20}$$

and symbol estimation at the second iteration is obtained as:

$$\begin{aligned}\hat{\mathbf{S}}^{(1)^2}(m) &= \frac{\mathbf{\Lambda}_l^{(1)^H}(m)\hat{\mathbf{Q}}_l^{(1)^1}(m)}{|\mathbf{\Omega}_{1,l}^{(1)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(1)}(m)|^2}\\ &= \mathbf{S}^{(1)}(m)+\frac{1}{4}\mathbf{\Gamma}_l^{(1)}(m).\end{aligned}\tag{21}$$

Finally, symbol estimation for the p^{th} iteration of EM algorithm, is obtained as:

$$\hat{\mathbf{S}}^{(1)^p}(m)=\mathbf{S}^{(1)}(m)+\frac{1}{2^p}\mathbf{\Gamma}_l^{(1)}(m).\tag{22}$$

It is considered that, with increasing the number of iteration of EM algorithm, the effect of noise at the symbol detection are decreased and error of symbol estimation is approached to zero.

B. Three users

At the first iteration of EM algorithm, can be obtained:

$$\begin{aligned}\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)^0}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)^0}(m)\\ &+\mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)^0}(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)\\ &+\mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m)+3\eta_l(m)\end{aligned}\tag{23}$$

According to equation (6) for M=3 we obtain:

$$\begin{aligned}\mathbf{Q}_l(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)\\ &+\mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m)+\eta_l(m)\end{aligned}\tag{24}$$

$\beta^{(i)}=\frac{1}{3}$ for all three users thus $\beta^{(i)}\Big(\mathbf{Q}_l(m)-\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m)\Big)=-\frac{2}{3}\eta_l(m)$.

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(1)^0}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\eta_l(m)-\frac{2}{3}\eta_l(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\frac{1}{3}\eta_l(m)\end{aligned}\tag{25}$$

we have for first iteration:

$$\begin{aligned}\hat{\mathbf{S}}^{(1)^1}(m) &= \frac{\mathbf{\Lambda}_l^{(1)^H}(m)\hat{\mathbf{Q}}_l^{(1)^0}(m)}{|\mathbf{\Omega}_{1,l}^{(1)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(1)}(m)|^2}\\ &= \mathbf{S}^{(1)}(m)+\frac{1}{3}\mathbf{\Gamma}_l^{(1)}(m).\end{aligned}\tag{26}$$

At the second iteration of EM algorithm, we have:

$$\begin{aligned}\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^1}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)^1}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)^0}(m)\\ &+\mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)^0}(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m)\\ &+\mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m)+\frac{7}{3}\eta_l(m)\end{aligned}\tag{27}$$

The $\mathbf{Q}_l(m)$ is constant and is obtained from equation (24).

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(1)^1}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)+\frac{1}{3}\eta_l(m)-\frac{4}{9}\eta_l(m)\\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m)-\frac{1}{9}\eta_l(m)\end{aligned}\tag{28}$$

$$\begin{aligned}\hat{\mathbf{S}}^{(1)^2}(m) &= \frac{\mathbf{\Lambda}_l^{(1)^H}(m)\hat{\mathbf{Q}}_l^{(1)^0}(m)}{|\mathbf{\Omega}_{1,l}^{(1)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(1)}(m)|^2}\\ &= \mathbf{S}^{(1)}(m)-\frac{1}{9}\mathbf{\Gamma}_l^{(1)}(m).\end{aligned}\tag{29}$$

With the same manner, we obtain:

$$\hat{\mathbf{S}}^{(1)^3}(m)=\mathbf{S}^{(1)}(m)-\frac{11}{27}\mathbf{\Gamma}_l^{(1)}(m).\tag{30}$$

$$\hat{\mathbf{S}}^{(1)^4}(m)=\mathbf{S}^{(1)}(m)-\frac{11}{27}\mathbf{\Gamma}_l^{(1)}(m).\tag{31}$$

If symbol estimation for the $(p-1)^{th}$ iteration of EM algorithm is considered as:

$$\hat{\mathbf{S}}^{(1)^{p-1}}(m)=\mathbf{S}^{(1)}(m)+\frac{a}{b}\mathbf{\Gamma}_l^{(1)}(m).\tag{32}$$

Then symbol estimation for the p^{th} iteration of EM algorithm is obtained as:

$$\hat{\mathbf{S}}^{(1)^p}(m) = \mathbf{S}^{(1)}(m) + (\frac{a}{b} - \frac{b+a}{3\times b})\mathbf{\Gamma}_l^{(1)}(m). \quad (33)$$

According to the equation (33), the accuracy of symbol estimation decrease with increasing the iteration of EM algorithm and it is not favorite. For solving this problem, after the computation of $\hat{\mathbf{S}}^{(1)^1}(m)$, first we obtain $\hat{\mathbf{S}}^{(2)^1}(m)$ and $\hat{\mathbf{S}}^{(3)^1}(m)$ and then go to second iteration of EM algorithm. This method do for all iteration of EM algorithm that several iteration is computed as follows. $\hat{\mathbf{S}}^{(1)^1}(m)$ is obtained according to equation (26). For the second step according to equation (27), can be written:

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(2)^0}(m) &= \mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m) + \eta_l(m) - \frac{4}{9}\eta_l(m) \\ &= \mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m) + \frac{5}{9}\eta_l(m) \quad (34)\end{aligned}$$

Thus first symbol estimation for second user is obtained as:

$$\begin{aligned}\hat{\mathbf{S}}^{(2)^1}(m) &= \frac{\mathbf{\Lambda}_l^{(2)^H}(m)\hat{\mathbf{Q}}_l^{(2)^0}(m)}{|\mathbf{\Omega}_{1,l}^{(2)}(m)|^2 + |\mathbf{\Omega}_{2,l}^{(2)}(m)|^2} \\ &= \mathbf{S}^{(2)}(m) + \frac{5}{9}\mathbf{\Gamma}_l^{(2)}(m). \quad (35)\end{aligned}$$

At the third step we have:

$$\begin{aligned}\sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m) &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)^1}(m) + \mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)^1}(m) \\ &+ \mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)^0}(m) \\ &= \mathbf{\Lambda}_l^{(1)}(m)\mathbf{S}^{(1)}(m) + \mathbf{\Lambda}_l^{(2)}(m)\mathbf{S}^{(2)}(m) \\ &+ \mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m) + \frac{17}{9}\eta_l(m) \quad (36)\end{aligned}$$

thus $\beta^{(i)}\Big(\mathbf{Q}_l(m) - \sum_{j=1}^{M}\mathbf{Z}_l^{(j)^0}(m)\Big) = -\frac{8}{27}\eta_l(m)$.

$$\begin{aligned}\hat{\mathbf{Q}}_l^{(3)^0}(m) &= \mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m) + \eta_l(m) - \frac{8}{27}\eta_l(m) \\ &= \mathbf{\Lambda}_l^{(3)}(m)\mathbf{S}^{(3)}(m) + \frac{19}{27}\eta_l(m) \quad (37)\end{aligned}$$

Thus first symbol estimation for third user is obtained as:

$$\hat{\mathbf{S}}^{(3)^1}(m) = \mathbf{S}^{(3)}(m) + \frac{19}{27}\mathbf{\Gamma}_l^{(3)}(m). \quad (38)$$

With the same manner, symbol detection for the second iteration of EM algorithm are obtained as:

$$\hat{\mathbf{S}}^{(1)^2}(m) = \mathbf{S}^{(1)}(m) + \frac{11}{81}\mathbf{\Gamma}_l^{(1)}(m). \quad (39)$$

$$\hat{\mathbf{S}}^{(2)^2}(m) = \mathbf{S}^{(2)}(m) + \frac{103}{243}\mathbf{\Gamma}_l^{(2)}(m). \quad (40)$$

$$\hat{\mathbf{S}}^{(3)^2}(m) = \mathbf{S}^{(3)}(m) + \frac{449}{729}\mathbf{\Gamma}_l^{(3)}(m). \quad (41)$$

According to the equation (39)-(41), it is considered that the accuracy of symbol estimation increase with increasing the iteration of EM algorithm and it is favorite.

V. Simulation Results

In our simulations, we consider a system with two and three users that each user uses two antennas to transmit signal according to Alamouti scheme. The receiver has one antenna. The FFT size is considered equal to 64. At the simulations, QPSK modulation is used and the results are averaged over 2000 different symbol realizations.

The channels between each transmit and receive antennas are assumed to be independent. The bit error rate for the different number of iteration of EM algorithm is shown in table 1 for two users. According to table 1, it is considered that the value of the bit error rate at the third iteration of EM algorithm equals to zero that this value is a excellent result. Fig.2 show the bit error rate for the different number of iteration of EM algorithm for three users. According to Fig.2, the bit error rate decreases until the 4th iteration and after that, it remains almost without change because at the high iteration, symbol estimation remains almost constant according to three users subsection from section IV. Fig.3 show the bit error rate for the different number of received antenna for three users. According to Fig.3, the bit error rate decreases with increasing the number of received antenna. Signal to interference plus noise ratio for three users is showed in Fig.4. Signal to noise ratio and Signal to interference plus noise ratio are obtained as [9]:

$$SNR = \frac{(|\mathbf{\Omega}_{1,l}^{(1)}(m)|^2 + |\mathbf{\Omega}_{2,l}^{(1)}(m)|^2)R_{X^{(1)}}}{R_n} \quad (42)$$

$$SINR^i = \frac{(|\mathbf{\Omega}_{1,l}^{(i)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(i)}(m)|^2)R_{X^{(i)}}}{\sum_{j=1,j\neq i}^{M}(|\mathbf{\Omega}_{1,l}^{(i)}(m)|^2+|\mathbf{\Omega}_{2,l}^{(i)}(m)|^2)R_{X^{(i)}}+R_n} \quad (43)$$

where $\Big[\mathbf{\Lambda}_l^{(i)^H}(m)\mathbf{\Lambda}_l^{(i)}(m)\Big] = (|\mathbf{\Omega}_{1,l}^{(i)}(m)|^2 + |\mathbf{\Omega}_{2,l}^{(i)}(m)|^2)I$, $R_{X^{(i)}} = E[S^{(i)}(m)^H S^{(i)}(m)]$, $R_n = E[\eta_l(m)^H\eta_l(m)]$. At Fig.5, the bit error rate of EM-based symbol detection algorithm is compared with probabilistic data association (PDA) symbol detection algorithm [10] for three users. In this simulation, we suppose that the receiver has two antennas. According to Fig.5, it is considered that the bit error rate proposed algorithm is lower than PDA algorithm.

TABLE I

THE BIT ERROR RATE FOR TWO USERS

SNR(dB) / **Iteration**	-15	-10	-5	0	5	10	15
1	0.4157	0.3392	0.2279	0.1180	0.0435	0.0107	0.0018
2	0.3198	0.2046	0.0974	0.0341	0.0077	0.0015	0.0001
3	0.1791	0.0810	0.0264	0.0056	0.0009	0.0001	0
4	0.0668	0.0188	0.0038	0.0006	0.0002	0	0
8	3×10^{-5}	2×10^{-6}	0	0	0	0	0
10	0	0	0	0	0	0	0

VI. Conclusion

In this paper we propose an EM-based symbol detection algorithm for space time block coded MIMO-OFDM systems.

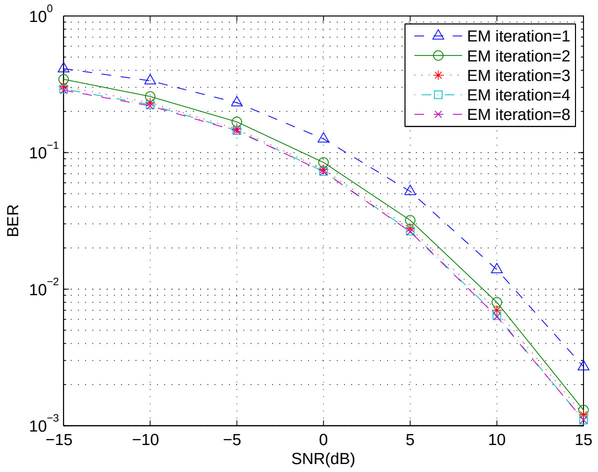

Fig. 2. BER for three users, FFT size=64, NR=1

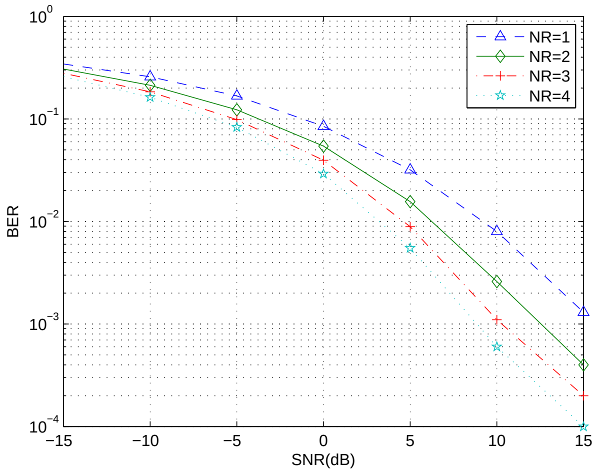

Fig. 3. BER for different receiver antenna, FFT size=64, M=3

The proposed algorithm is based on parameter estimation of superimposed signals using the Expectation Maximization technique. Simulation results show that the bit error rate significantly is decreased with increasing number of iteration of EM algorithm.

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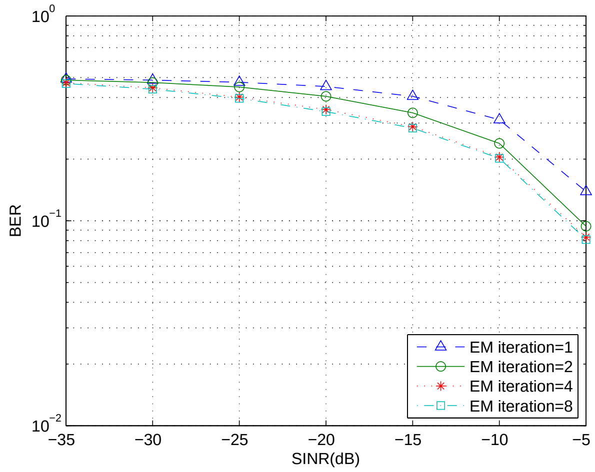

Fig. 4. BER for three users, FFT size=64, NR=1

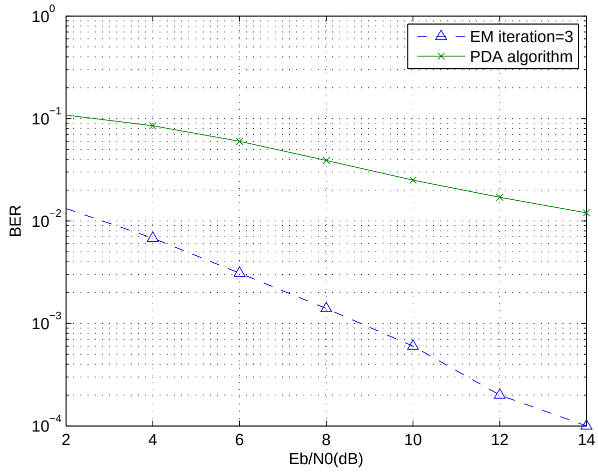

Fig. 5. comparison of different algorithm, FFT size=64, M=3, NR=2

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