Channel Estimation in OFDM System Based on the Linear Interpolation, FFT and Decision Feedback

First R. Hajizadeh, Second K. Mohamedpor, and Third M.R. Tarihi

***Abstract* — In this paper, we propose a new channel estimation algorithm based on the training data for orthogonal frequency division multiplexing (OFDM) system in fast fading channel. In this method, channel frequency response (CFR) is estimated. In the first step, channel response is estimated at pilot subcarriers by last square method, and then primary whole CFR is obtained by linear interpolation. In the second step, the effect of noise on estimated channel is reduced by using FFT processing and then transmitted data is detected. For increasing accuracy of channel estimation, detected data is modulated again. Then by using received signal in receiver and new modulated data, CFR is estimated anew and is passed from noise reducing algorithm. And finally, data is detected again. This process is done about three or four times to reduce the effect of noise adequately. Computer simulations demonstrate that proposed method improves bit error rate (BER) and mean square error (MSE) compared to usual estimation methods.**

***Keywords* — Channel Estimation, Decision Feedback, FFT, OFDM.**

I. INTRODUCTION

WIRELESS communication and transmitting data have many demands these days and are used in many applications. Wireless communications have many problems and restrictions. Channel estimation and destruction by channel, power limitation of transmitted data, complexity of mobile receivers, band width limitation that should be distributed between some users, are some of these problems. Among of wireless techniques, orthogonal frequency division multiplexing (OFDM) has many advantages and especially is noticed in frequency selective Rayleigh channels. OFDM is a general technique in signal transmission on the wireless communication. This technique converts a frequency selective channel to a collection of flat frequency selective sub-channel that leads to simplicity of receiver structure [1]. In these systems, inter-symbol interference (ISI) can be removed by using of cyclic prefix (CP). Cyclic prefix is inserted between consecutive transmitted blocks and should be increased beyond the maximum delay expansion of channel. A good delineation of this parameter leads to simpler channel synchronization and the effect of channel appears as a simpler scalar multiplication in the frequency domain. Hence each subcarrier is attenuated by the corresponding narrowband sub-channel coefficient. We use this property to estimate channel in frequency domain.

In OFDM systems, detection of transmitted data and capacity of system highly depend on channel state information (CSI) in transmitter and receiver. In other word coding, modulation and interference prevention methods can be applied in these systems by knowledge of channel characteristics. If channel is estimated accurately, undesirable effects on channel response will be compensated.

In recent years, several channel estimation methods are proposed. Considering of these methods shows that achieving more accuracy leads to more complexity. So it seems necessary to propose a simple and precise method. The CSI can be obtained through two methods: 1) blind channel estimation 2) training-based channel estimation.

Blind channel estimation explores the statistical information of channel and certain properties of transmitted signal and needs a large amount of data. Hence this method isn't proper in fast fading channels. Training-based channel estimation is based on the training data, which is inserted into main data, sent at the transmitter and known a priori at the receiver [2].

In this paper, training-based channel estimation is applied. Pilot symbols, which is known by transmitter and receiver, is used for initial estimation or training. For inserting the pilot in OFDM symbols, two major methods were proposed: Block-type and Comb-type, which illustrated in figure 1 [3].

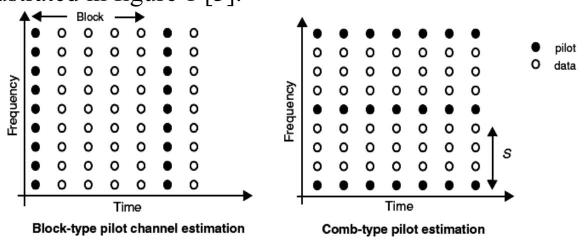

Fig. 1. Two Basic Types of Pilot Arrangement for OFDM Channel Estimations

Comb-type pilots initialization has considered for fast channel variation and Block-type is proper for slow channel variation [4, 5, 6].

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In training-based channel estimation, two usual algorithms exist to obtain channel coefficients in pilot subcarriers: 1) minimum mean square error (MMSE), 2) Least Square (LS) [7].

Least square has lower complexity and is simple and doesn't need to statistical information of channel and noise; but its accuracy is lower than MMSE in frequency selective channels. MMSE method is resistant against Doppler effects in multipath channels, but needs to calculate covariance matrix and inverse matrix and statistical information of noise should be known at receiver; hence its complexity is high [8]. In this paper, lower complexity is important for us, so we use LS method.

In this paper, channel frequency response (CFR) is estimated for OFDM systems in fast fading channels. At first channel coefficients in pilot subcarriers are calculated by using LS algorithm. Then whole CFR is obtained by linear interpolation. In next step, the noise effect on the estimated channel is reduced by using discrete Fourier transform (DFT) processing and transmitted data is detected. Then for enhancing accuracy of channel estimation, detected data is modulated in receiver again, and channel is estimated by using new modulated data and received signal and is passed from noise reducing algorithm anew. And finally transmitted data is detected again. This processing has to be repeated three or four times to reduce the effect of noise adequately. Computer simulations show that proposed method improved performance compared to Spline and Gaussian radial basis function network (GRBFN) interpolation and if we use Spline or GRBFN interpolation instead of linear interpolation in this method, performance of algorithm doesn't change and their results are similar whereas linear interpolation has the lowest complexity.

The rest of this paper is organized as follow. In section II, we briefly describe OFDM systems and channel model. Then in section III, proposed algorithm is presented. Finally we show simulation results in section IV.

II. OFDM SYSTEM

In OFDM systems, data is transmitted on narrow-band subcarriers in frequency domain. Figure 2 shows some of these subcarriers in frequency domain. Sub-carriers have overlap in frequency domain, hence frequency efficiency is increased. If subcarriers are completely orthogonal, inter-channel interference (ICI) can be removed.

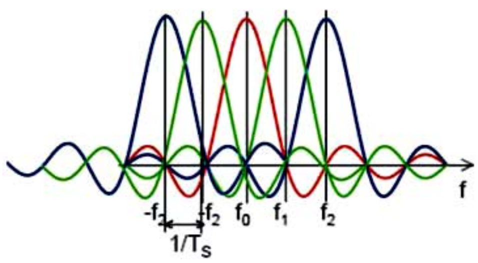

Fig. 2. Sub-carriers in an OFDM system.

Fig 3 shows a block diagram of an OFDM system.

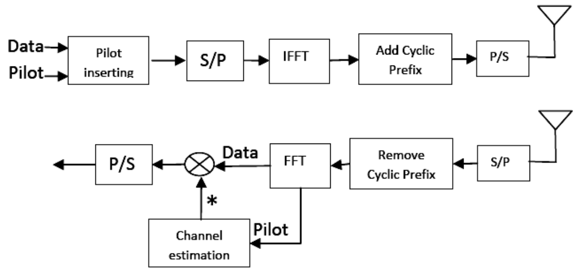

Fig. 3. Block diagram of an OFDM system [1]

In this system, after pilot insertion between data sequence at the transmitter, the result data is modulated by inverse discrete Fourier transform (IDFT) on N parallel subcarriers and then after receiving signal at receiver transformed back to frequency domain by DFT. In fact, IDFT converts frequency domain data into time domain. The number of points of the IDFT/DFT is equal to the total number of sub-carriers. Every subcarrier can be formulated as follow:

$$S_c(t) = A_c(t) e^{j[\omega_c t + \Phi_c(t)]} \tag{1}$$

Where, $A_c(t)$ is amplitude and $\Phi_c(t)$ is phase. An OFDM signal is constructed from some of these subcarriers, so it can be described as follow:

$$S_s(t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n(t) e^{j[\omega_n t + \Phi_n(t)]} \tag{2}$$

$$\omega_n = \omega_0 + n\Delta\omega$$

$A_c(t)$ and $\Phi_c(t)$ get different values in different symbols, but they are constant in every symbol and only depend on frequency of carriers. It means that we have in every symbol:

$$\Phi_n(t) \Rightarrow \Phi_n \;,\; A_n(t) \Rightarrow A_n \tag{3}$$

If signal is sampled with 1/T (T is duration of a symbol) and equation (3) is inserted into equation (2), we will have:

$$S_s(kT) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j[(\omega_n + n\Delta\omega)kT + \Phi_n]} \tag{4}$$

It is obvious that with $\omega_0 = 0$, equation (3) is converted to an IDFT transform. Therefore OFDM modulation is an IDFT transform inherently.

In continuation cyclic prefix is inserted. Cyclic prefix is a crucial feature of OFDM that is used to prevent the inter-symbol interference (ISI) and inter-channel interference (ICI). ISI and ICI are produced by the multi-path channel through which the signal in propagated. Cyclic prefix protects orthogonality between sub-channels. The duration of the cyclic prefix should be longer than the maximum delay spread of the multi-path environment.

$$x_T(n) = \begin{cases} x(N+n), n = -N_{cp}, -N_{cp}+1, \ldots, -1 \\ x(n), n = 0, 1, \ldots, N-1 \end{cases} \tag{5}$$

For adding cyclic prefix, a part of the end of the OFDM time-domain waveform is added to the front of it. Cyclic prefix is caused that circular convolution is converted to linear convolution. Therefore the effect the channel on each subcarrier can be presented by a single complex multiplier that is shown in equation (6):

$$Y(k) = S(k)H(k) + W(k) \qquad (6)$$

Where, $H(k)$ is the Fourier transform of channel impulse response (CIR). The frequency selective channel is modeled as a finite impulse response (FIR) filter.

$$h(n) = \sum_{i=0}^{L} g_i \delta(n - \lambda_i) \qquad (7)$$

L is number of path and g_i is the channel gain in ith path and is independent complex Gaussian random process with zero mean and unit variance, and λ_i is the delay of the ith path. Therefore,

$$H(k) = FFT\{h(n)\} = \frac{1}{N}\sum_{n=0}^{N-1} h(n) e^{-j(2k\pi n/N)} \qquad (8)$$
$$k = 0,1,\ldots,N-1$$

Transmitted data, after passing through the channel and adding noise, is received as follow:

$$y(n) = s(n) \otimes h(n) + \omega(n) \qquad (9)$$

Where, y is received signal, s is transmitted data, and ω is additive white Gaussian noise. Equation (10) shows received signal after removing cyclic prefix and applying FFT on it.

$$Y(k) = S(k)H(k) + W(k) \qquad (10)$$

That *W* and *H* are the Fourier transform of the noise and h respectively.

In continuation, the channel is estimated in pilot subcarriers, and then whole channel frequency response is obtained by interpolation. And finally, data is detected as follow:

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)} \qquad (11)$$

Where, $Y(k)$ is Fourier transform of $y(n)$. $\hat{X}(k)$ and $\hat{H}(k)$ are transmitted data and estimated channel respectively.

III. Proposed Channel Estimation Algorithm

In this paper, we use training-based channel estimation with comb-type pilot arrangement. At first, channel coefficients are obtained in pilot subcarriers and then whole channel response is estimated by interpolation between pilot subcarriers. Block diagram of proposed method is briefly shown in figure 4.

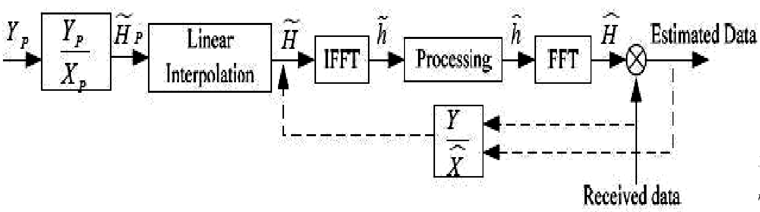

Fig. 4. Channel estimation structure block diagram

A. Step 1: Channel Estimation in Pilot Subcarriers and Obtaining of CFR by Linear Interpolation.

After removing cyclic prefix and getting DFT from received signal, channel coefficients in pilot subcarriers are estimated by using least square algorithm.

$$\hat{H}_P(k) = \frac{Y_P(k)}{X_P(k)}, \; k \in pilot\_index \qquad (12)$$

Then whole channel frequency response is obtained in total subcarriers by linear interpolation.

$$\hat{H}_p \xrightarrow{Linear\ Interpolation} \hat{H} \qquad (13)$$

B. Step2: FFT processing for reducing noise effect.

There is not enough pilot subcarriers at the edges of OFDM symbols in training-based channel estimation, therefore estimated coefficients in the edge subcarriers have lower precise. Another problem is effect of noise on the pilot subcarriers. Reference [9] proposes an algorithm based on the FFT processing to reduce these effects. We apply this method with a little change and use this processing with a linear interpolation. Simulations demonstrate our proposed algorithm has similar results whereas has lower complexity because of linear estimator. In this algorithm, channel impulse response (CIR) is studied. Figure 5 shows a typical magnitude of CIR.

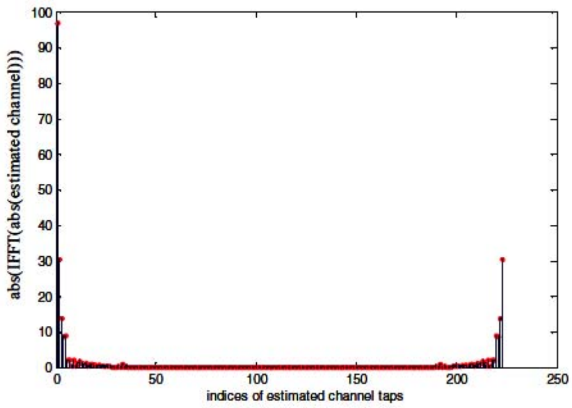

Fig. 5. Typical magnitude of CIR

It is obvious that most of energy concentrate into a subset of taps. By zeroing the taps out of this subset, only major taps remain and the effect of noise is reduced. We use below algorithm to determine taps which should be zero [9].

At first, IFFT of $|\hat{H}|$ is calculated that is symmetric.

$$e(n) = \left| \sum_{k=0}^{N_s-1} \left|\hat{H}(k)\right| e^{j\left(2k\pi n/N_s\right)} \right|, n = 0,1,\ldots,N_s - 1 \quad (14)$$

Then total energy of taps computed.

$$E_{total} = 2 \sum_{n=1}^{N_s/2} e(n) \qquad (15)$$

Now we calculate taps magnitude from one until the m of magnitude of the selected taps is equal to %40 erall, because IFFT of $|\hat{H}|$ is symmetric.

$$i_{th} = 1;$$
$$E_{th} = 0;$$
$$While\ E_{th} < 0.4 * E_{total}$$
$$E_{th} = 2 * Sum\left(e\left(i_{th}\right)\right) + E_{th}; \quad (16)$$
$$i_{th} = i_{th} + 1;$$
$$End;$$
$$i_{th} = i_{th} - 1;$$

Therefore $1,...,i_{th}$ and $i_{th},...,N_s$ are indices of taps with higher energy. Now $\tilde{h}(n)$ is modified.

$$\tilde{h}(n) = IFFT\left\{\tilde{H}(k)\right\} = \sum_{k=0}^{N_s-1} \tilde{H}(k) e^{j\left(2k\pi n / N_s\right)} \quad (17)$$

$$\hat{h} = \begin{cases} \tilde{h}(n), n \le i_{th}, n \ge N_s - i_{th} + 1 \\ 0, ohterwise \end{cases}$$

And finally,

$$\tilde{H}(k) = FFT\left\{\hat{h}(n)\right\} = \frac{1}{N_s} \sum_{n=0}^{N_s-1} \hat{h}(n) e^{-j\left(2k\pi n / N_s\right)} \quad (18)$$
$$k = 0,1,...,N_s - 1$$

$\tilde{H}$ is CFR after passing $\tilde{H}$ through FFT processing. Then data is detected by using $\tilde{H}$ and received signal by below equation.

$$\tilde{X}(k) = \frac{Y(k)}{\tilde{H}(k)} \quad (19)$$

Computer simulations show this step of algorithm is very effective in improving performance.

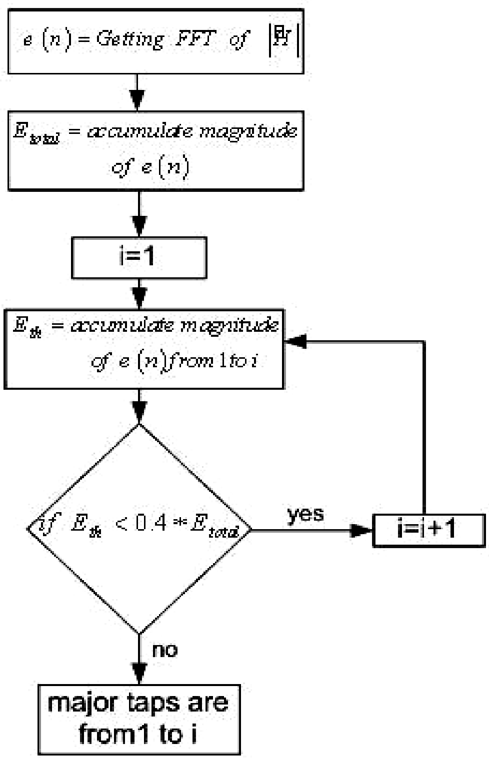

Fig. 6. Flowchart of determination of major taps.

Step 3: Using Iterative decision feedback technique.

We use an iterative decision feedback technique to improve performance more and more. In this technique, data is modulated again in receiver and then by using received signal, CFR is obtained again as follow equation:

$$\tilde{H} = \frac{Y}{\tilde{X}} \quad (20)$$

In next stage, achieved CFR is passed through FFT processing algorithm. And then data is detected by using new achieved CFR.

This operation is repeated 4 times. Last $\tilde{H}$ is the best CFR and computer simulations demonstrate that proposed algorithm is reduced the effect of noise adequately and improves performance compared to the conventional methods.

IV. Simulation Results

In this paper, simulations are performed for an OFDM system with 256 subcarriers. Data modulation is QAM and comb-type pilot arrangement is used.

The number of pilot subcarriers per OFDM is 32 and cyclic prefix length is 32. Channel coefficients in pilot subcarriers are estimated by using LS algorithm and channel interpolation is performed by linear interpolation. The studied channel model is frequency selective Rayleigh model with 5 paths and channel changes per every OFDM symbol.

Proposed algorithm is used to reduce the noise effect on channel estimation and improve performance. This method is compared with Spline interpolator and GRBFN interpolator.

For evaluating of performance of proposed channel estimator, mean square error (MSE) criteria based on estimation errors versus SNR are presented in figure 7. In this figure, the comparison of several channel estimation methods is shown: LS estimation with Spline interpolation, LS estimation with GRBFN interpolation, LS estimation with linear interpolation and applying step 2 of proposed algorithm and our proposed algorithm. Simulation results demonstrate that performance of proposed algorithm is better than the other mentioned methods.

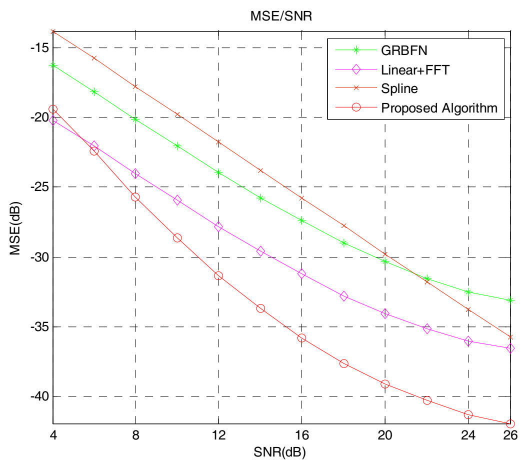

Fig. 7. MSE values of the channel estimators versus SNR.

In another performance evaluation for channel estimator, bit error rate (BER) criteria based on estimation errors versus SNR are presented in figure 8. This figure shows the comparison of mentioned methods and demonstrates that proposed algorithm is better than other method.

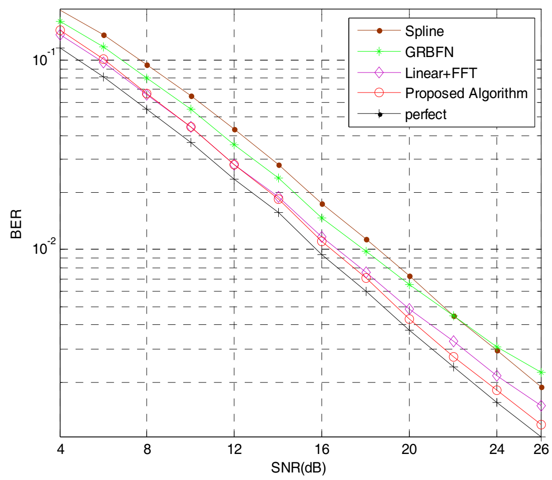

Fig. 8. BER values of the channel estimators versus SNR.

We use Spline and GRBFN interpolation instead of linear interpolation in proposed algorithm and their results are compared with our proposed algorithm in figure 9 and 10. Simulations show that all of their results are completely similar. Hence our proposed algorithm improves performance and has lower complexity than other methods because of linear interpolator.

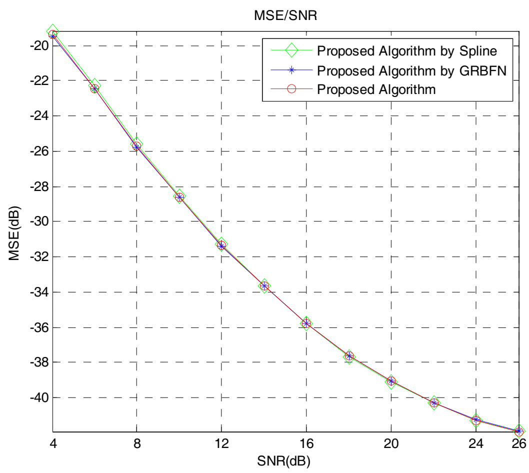

Fig. 9. Compared MSE of proposed algorithm with applying different interpolator.

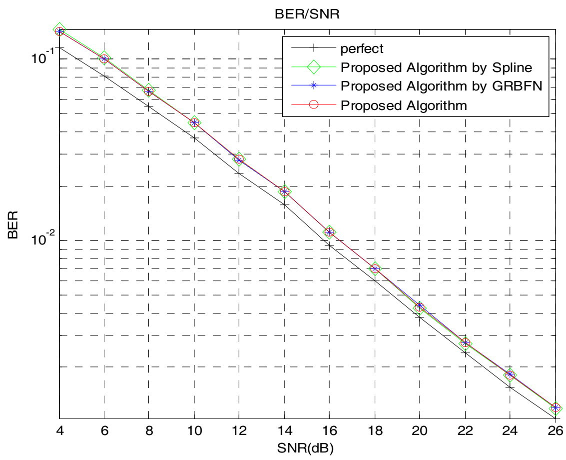

Fig. 10. Compared BER of proposed algorithm with applying different interpolator.

V. CONCLUSION

In this paper we proposed a new channel estimation algorithm based on the FFT processing and decision feedback for OFDM systems at fast fading selective frequency channel. This proposed algorithm in compared to existing channel estimation methods offers lower complexity because of linear interpolator whereas it has better performance. Therefore, with doing some simple processing on data can be reduced estimation error and complexity. It is obvious that proposed algorithm has lower complexity but considering numerical of complexity can show a certain and it is proposed for future works.

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