Spreading Sequence Adaptation for Dynamic MC-CDMA Systems

Seyed Mohsen Beheshti, Behrad Mahboobi, and Mehrdad Ardebilipour

***Abstract* —In this work we consider the uplink scenario of a single cell multi carrier code division multiple access (MC-CDMA) in which the channel between a given user and the base station receiver is assumed known and stable for the duration of the transmission. We propose an algorithm that uses the greedy interference avoidance method as well as power update to adapt the transmitter waveform of users. Then we illustrate the algorithms with examples.**

***Keywords* — Interference Avoidanc, waveform adaptation, MC-CDMA, signature sequence, power control.**

I. Introduction

THE high data rate communication systems require modulation techniques that improve the band efficiency and system robustness against fading. One of the modulation methods that can be used to accomplish these demands is MC-CDMA [1]. In MC-CDMA, instead of applying spreading sequences in the time domain, we can apply them in the frequency domain, mapping a different chip of a spreading sequence to an individual OFDM subcarrier. Hence each OFDM sub carrier has a data rate identical to the original input data rate and the multi carrier system “absorbs” the increased rate due to spreading in a wider frequency band [2].

For a multiple access channel, the main limiting factor is the multiple access interference (MAI). Several works performed to overcome the effect of MAI in a multiple access system. These works falls under two major areas: optimization of transmitted power and signal design [3]. In this paper we use the adaptive greedy interference avoidance approach and power allocation jointly, to mitigate the interference seen by each user at corresponding receiver in the base station subject to the constraint on SINR and received power for each user. An algorithm for constructing optimal spreading codes and powers in DS-CDMA is presented in [4,5]. Another adaptive algorithm based on distributed noncooperative game theory under the ideal channel (slow and frequency non selective fading channel with unit gain) is proposed in [6] that converges to spreading codes and powers that is identical to optimal solution derived in [4,5]. Greedy interference avoidance algorithm is applied to uplink CDMA dispersive channel with fixed and equal power for all users [7]. In this case no constraint considered except unity norm of signature sequences. It is shown that, sum capacity of channel monotonically increases and hence, the codeword ensemble derived from algorithm, maximizes the sum capacity for given set of user powers. Very recently, a game theoretic algorithm based on [6] proposed to joint transmitter adaptation and power control [8].

Our goal is to generalize the algorithm presented in [6-8]. In our new scenario, each carrier of a given user experiences a channel gain which is different from other carriers. Furthermore, we consider an additional constraint on the received power of each user. We will show that the derived spreading sequence ensemble and power set are the well known generalized Welch bound equality (GWBE) sequences.

The paper organized as follows. We describe the system model and problem statement in Section II. In section III we present the update rules based on interference avoidance algorithm for joint signature sequence and power adaptation. Then we propose our algorithm in section IV. In section V we perform the simulation results of the algorithm.

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II. System Model And Problem Statement

A. System Model

We consider the uplink of a synchronous MC-CDMA system, where there are K simultaneous users that spread data on N orthogonal subcarriers using different spreading codes. The transmitted signal of the data symbol of the k th user, $s_k(t)$ $k=1,...,K$ in j the pulse duration is

$$s_k(t)=\sum_{i=1}^{N}\sqrt{p_k}\,b_k\,c_k^i\,e^{2\pi(f_0+if_d)t}\,p(t-jT), \qquad (1)$$

where b_k and p_k are the symbol transmitted by k th user and its power, respectively. c_k^i for $i=1,...,N$ is the i th chip of the spreading sequence corresponding to user k and f_d is the subcarrier separation. $p(t)$ in equation (1) is a time shifted rectangular signaling pulse, given by

$$p(t)=\begin{cases}1 & \mathit{for}\; 0\le t\le T\\ 0 & \mathit{otherwise.}\end{cases} \qquad (2)$$

The received signal corresponding to k th user is

$$r_k(t)=\sum_{i=1}^{N}h_k^i\,\sqrt{p_k}\,b_k\,c_k^i\,e^{2\pi(f_0+if_d)(t-t_d)}\,p(t-jT-t_d), \quad (3)$$

where h_k^i is the frequency response of the channel to i th subcarrier of k th user and t_d is the time delay. Hence, the received signal can be expressed as:

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t), \tag{4}$$

where $n(t)$ is the additive white Gaussian noise at the receiver. Let us represent the vector of MC-CDMA spreading code as $\mathbf{c}_i$, the signature sequence of user k. Hence, equation (4) can be written as

$$\mathbf{r} = \sum_{k=1}^{K} \sqrt{p_k}\, b_k\, H_k\, \mathbf{c}_k + \mathbf{n}, \tag{5}$$

where $H_k = diag(h_k^1, h_k^2, ..., h_k^N)$ and $\mathbf{c}_k = [c_k^1, c_k^2, ..., c_k^N]^T$ are the channel matrix and signature sequence of user k, respectively, and $\mathbf{n}$ is an independent and identically distributed (i.i.d.) Gaussian vector with covariance matrix $\mathbf{W} = \sigma^2 \mathbf{I}_N$, which is independent of the transmitted symbols. We note that all user signature sequences take values in the $N-$dimensional sphere with radius 1, i.e.

$$\mathbf{c}_k^T \mathbf{c}_k = 1 \qquad \forall\, k = 1, ..., K. \tag{6}$$

Also we consider a received power constraint $\overline{P}$ for all users. For the next analysis we define

$$\mathbf{c}_k^r = \frac{H_k \mathbf{c}_k}{\left\| H_k \mathbf{c}_k \right\|}, \tag{7}$$

and

$$p_k^r = p_k \left\| H_k \mathbf{c}_k \right\|^2, \tag{8}$$

as normalized received sequence and received power, respectively.

B. Structure of Optimum Linear Receiver

It is well known that the MMSE receiver is the optimum linear receiver, optimum in the sense of maximizing the SINR of each user. As mentioned in [4], MMSE receiver and matched filter (MF) show the same performance at the optimal solution. Thus we can assume that matched filters are employed at the receiver to detect users, where the decision variable d_k for user k is

$$d_k = (\mathbf{c}_k^r)^T \mathbf{r} = (\mathbf{c}_k^r)^T \left(\sqrt{p_k^r} b_k \mathbf{c}_k^r + \sum_{\substack{i=1 \\ i \neq k}}^{K} \sqrt{p_i^r} b_i \mathbf{c}_i^r + \mathbf{n} \right). \tag{9}$$

Thus, the SINR of user k becomes:

$$\gamma_k^{MF} = \frac{p_k^r}{(\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r)}, \tag{10}$$

where

$$\mathbf{R}_k = \mathbf{R} - p_k H_k \mathbf{c}_k \mathbf{c}_k^T H_k, \tag{11}$$

is the interference of user k plus noise correlation matrix and

$$\mathbf{R} = E\left[\mathbf{r}\mathbf{r}^T \right] = \sum_{k=1}^{K} p_k H_k \mathbf{c}_k \mathbf{c}_k^T H_k + \mathbf{W} \tag{12}$$

is the correlation matrix of received signal. We define the interference function of user k as denominator of k th user SINR

$$i_k = (\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r). \tag{13}$$

C. Problem Statement

As mentioned before, in our setup individual users adjust their signature sequences and powers to meet a set of specified target SINRs $\{\gamma_1^*, ..., \gamma_k^*, ..., \gamma_K^*\}$ with minimum transmitted power and signature sequences that minimize the interference seen by each user. But for this reason, the admissibility condition must be satisfied. Admissibility condition is presented for special case in which all of the users have the same target SINR [4]. After a few mathematical manipulations, we can generalize the admissibility condition as follows:

Definition: K users (each having target SINR γ_k^* $k = 1, ..., K$) are admissible in the system with processing gain N and received power constraint $\overline{P}$ if and only if

$$\sum_{k=1}^{K} e\left(\gamma_k^*\right) < N \frac{K\,\overline{P}}{K\,\overline{P} + N\,\sigma^2}. \tag{14}$$

where $e\left(\gamma_k^*\right) = \gamma_k^* / \left(\gamma_k^* + 1\right)$ is known as effective bandwidth. Note that admissibility condition without power constraint is an especial case of (14) ($\overline{P} \to \infty$).

Each user optimizes its signature sequence and power in two steps

1) Signature sequence Selection such that minimize its interference function for constant power. i.e

$$\min_{\mathbf{c}_k^r} i_k \qquad \text{subject to} \quad (\mathbf{c}_k^r)^T (\mathbf{c}_k^r) = 1, \tag{15}$$

2) power Adjustment to meet its target SINR. i.e.

$$p_k^r = \gamma_k^*\, i_k \tag{16}$$

III. Joint Signature Sequence And Power Adaptation

In this section we solve the problem stated in the previous section. It is easy to show that the problem (15) is convex. As mentioned in [6], this ensures that solving of problem (15) by individual users in each iteration converges to optimal solution. The solution is straightforward [6] and implies that the best strategy for user k is a greedy interference avoidance procedure in which user k's received signature sequence $\mathbf{c}_k^r$ is replaced by the minimum eigenvector of matrix $\mathbf{R}_k$ which minimizes the effective interference corrupting user k's signal at the receiver k [9].

We show that this choice of received signature sequence $\mathbf{c}_k^r$ is equivalent to replace the signature sequence of user k by the minimum eigenvector of matrix $H_k^{-1}\mathbf{R}_k H_k$.

Proof: As mentioned before, the solution of problem (15) is the minimum eigenvector of matrix $\mathbf{R}_k$, i.e.

$$\mathbf{R}_k \mathbf{c}_k^r = \lambda_k^{\min} \mathbf{c}_k^r, \tag{17}$$

where $\lambda_k^{\min}$ is the minimum eigenvalue of matrix $\mathbf{R}_k$. Using (7) and (17) we have

$$\mathbf{R}_k \frac{H_k \mathbf{c}_k}{\|H_k \mathbf{c}_k\|} = \lambda_k^{\min} \frac{H_k \mathbf{c}_k}{\|H_k \mathbf{c}_k\|}, \tag{18}$$

and hence

$$\left(H_k^{-1}\mathbf{R}_k H_k\right)\mathbf{c}_k = \lambda_k^{\min}\mathbf{c}_k. \tag{19}$$

It is worth mentioning that, the equality $\mathrm{eig}(\mathrm{AB}) = \mathrm{eig}(\mathrm{BA})$ implies that $\lambda_k^{\min}$ is the minimum eigenvalue of $H_k^{-1}\mathbf{R}_k H_k$, as well. □

Thus, in each signature sequence update we have:

$$\mathbf{c}_k(t+1) = \mathbf{x}_k(t), \tag{20}$$

where $\mathbf{x}_k(t)$ is the minimum eigenvector of $H_k^{-1}\mathbf{R}_k(t)H_k$.

Furthermore, using (8) and (16), power update for each user can be performed as follows

$$p_k(t+1) = \frac{\gamma_k^* i_k(t)}{\|H_k \mathbf{c}_k(t)\|^2} \tag{21}$$

However, applying (20) and (21) may lead to a new user signature sequences that are distant from the current user signature sequence in signal space, and/or abrupt power variation. This behavior is not desirable since it may lead to increase error probability at the receiver. Also, when connection loss between the transmitter and the base station occurs, base station is not able to adapt to these sudden changes. Thus, as in [6], the users change their signature sequence and powers in small increments, with corresponding incremental changes of the receiver filter that follow the transmitter changes.

We define the signature sequence update rule as

$$\mathbf{c}_k(t+1) = \frac{\mathbf{c}_k(t) + m\beta\mathbf{x}_k(t)}{\|\mathbf{c}_k(t) + m\beta\mathbf{x}_k(t)\|}, \tag{22}$$

where $m = \mathrm{sgn}(\mathbf{c}_k(t)^T \mathbf{x}_k(t))$ and β limits the Euclidean distance between two successive signature updates in signal space.

Also we define the power update rule as

$$p_k(t+1) = (1-\mu)p_k(t) + \mu\frac{\gamma_k^* i_k(t)}{\|H_k \mathbf{c}_k(t)\|^2} \tag{23}$$

where $0 < \mu < 1$.

IV. Dynamic Waveform Adaptation Algorithm

In this section we present our waveform adaptation algorithm by considering non ideal channel between user and base station, and received power constraint on each user. A formal statement of the algorithm is given below:

1) Initial setup: Start with randomly selected signature sequences, powers, and channel gain matrices.

2) Admissibility Check: IF the target SINRs and received power constraint satisfy (14) GO TO step 3 ELSE STOP: users can not be admissible.

3) Convergence Check: IF the difference between two successive convergence factor such as SINR are greater than ε GO TO step 4, ELSE STOP: an optimal configuration has been reached.

4) Adaptation Stage: FOR each user $k = 1,\dots,K$ DO

a) Evaluate current correlation matrix using equation (11) and then $H_k^{-1}\mathbf{R}_k(t)H_k$,

b) Determine the minimum eigenvector $\mathbf{x}_k(t)$,

c) Replace the current signature sequence using update equation (22),

d) Replace the current power using update equation (23).

5) GO TO step 3.

It has been shown that this algorithm converges to optimal fixed point stated in [4].

V. Simulation Results

In this section, we provide numerical examples to evaluate our algorithm. It has been shown that optimal received spreading codes are orthogonal to each other for $K \le N$ [4]. Thus, without loss of generality we assume that $K > N$. In our simulations, we consider the signal space dimension $N = 4$ and white noise with $\sigma^2 = 0.1$. The simulation constants are $\beta = \mu = 0.1$ and tolerance $\varepsilon = 10^{-5}$.

In our first experiment, we consider $K = 5$ users with target SINRs $\gamma^* = \{1.5, 2, 2.5, 3.5, 4\}$ and received power constraint $\bar{P} = 0.8$. Note that, the sum of effective bandwidths is equal to 3.5587<3.6364 and (14) is satisfied. We begin our simulation with randomly generated spreading codes and powers and following channel matrices.

$$H_1 = diag\{0.9473, 0.4624, 0.8473, 0.8418\}$$
$$H_2 = diag\{0.7371, 0.5105, 0.7583, 0.5800\}$$
$$H_3 = diag\{0.4805, 0.5276, 0.9370, 0.4429\}$$
$$H_4 = diag\{0.5455, 0.4323, 0.6650, 0.4080\}$$
$$H_5 = diag\{0.9383, 0.5180, 0.4560, 0.5844\}.$$

We define the $N \times K$ signature sequence matrix $\mathbf{C}$ in which the columns are the transmitted signature sequences by users. Our algorithm yields the matrix $\mathbf{C}$ and transmitted power vector as follows

$$\mathbf{C} = \begin{bmatrix} 0.5504 & -0.6162 & -0.2194 & -0.4031 & -0.0932 \\ -0.6267 & 0.6460 & -0.4806 & -0.8160 & -0.0620 \\ -0.3994 & -0.1616 & 0.0332 & 0.0785 & -0.9803 \\ -0.3805 & -0.4204 & -0.8484 & 0.4069 & 0.1628 \end{bmatrix}$$

$$\mathbf{P} = [\ 0.9833 \quad 1.7232 \quad 2.7538 \quad 3.5733 \quad 3.6968]$$

We compute received signature sequence and powers from (7) and (8), respectively and construct the $N \times K$ signature sequence matrix $\mathbf{C}^r = \left[\mathbf{c}_1^r, \ldots, \mathbf{c}_K^r\right]$ and received power vector $\mathbf{P}^r = \left[p_1^r, \ldots, p_K^r\right]$.

The weighted received signature sequence correlation matrix is:

$$(\mathbf{C}^r) diag(\mathbf{P}^r)(\mathbf{C}^r)^T = 0.8063\mathbf{I}_4$$

and is within $\mathcal{O}(10^{-4})$ tolerance from the corresponding matrix implied by [4]. This corresponds to a GWBE set.

We repeat the experiment once more time with received power constraint $\overline{P} = 2$ and following target SINRs

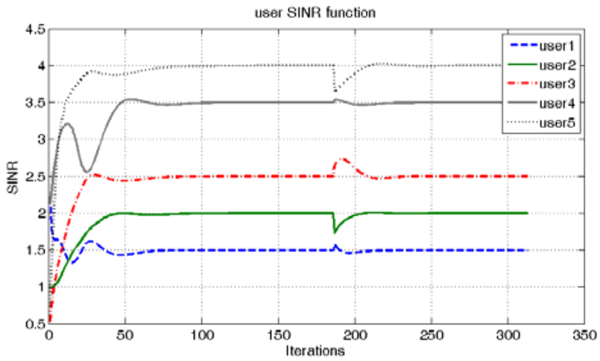
$$\gamma^* = \{1, 1.5, 2, 2.5, 10\}.$$

Note that, according to [4], the user 5 is oversized and the weighted received signature sequence correlation matrix is not scaled identity and hence, the signature sequence set is not a GWBE set. We also survey the cross-correlation matrix of user signature sequences

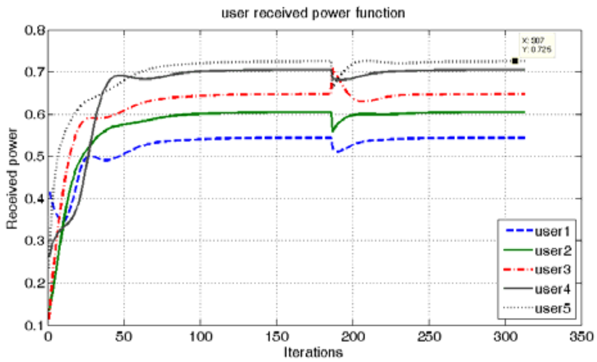
$$(\mathbf{C}^r)^T(\mathbf{C}^r) = \begin{bmatrix} 1.0000 & -0.4974 & 0.3966 & -0.3212 & 0 \\ -0.4974 & 1.0000 & 0.3016 & -0.2443 & 0 \\ 0.3966 & 0.3016 & 1.0000 & 0.1948 & 0 \\ -0.3212 & -0.2443 & 0.1948 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

This shows that oversized user 5 is orthogonal to the other non-oversized users.

We complete our work by illustrating the tracking ability of the algorithm, for fixed number of active users and slowly changing channel. We assume that a steady state configuration is reached. Then we change the channel gains randomly within $(0 \sim \pm 10)\%$ of their initial values. Fig. 1 shows that user SINRs, transmitted power and received powers vary smoothly during this channel variation. Note that convergence speed after channel variation is considerably less than convergence speed with randomly chosen channel.



(a)



(b)

Fig. 1. Channel variation tracking. (a) SINR variation. (b) Received power variation

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