# LMI Based Switching Congestion Controller for Packet Switching Networks

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Abstract — In this paper a new Linear Matrix Inequality (LMI) based switching controller for multiple Bottleneck packet switching Network has been considered. The main goal is to illustrate the effects of the Switching Control methodology on the congestion control problem of the packet switching Networks with dynamically varying parameters such as Link capacity and time delays. The congestion dynamic for congested network is presented and LMI based switching controller is being discussed. Then, the proposed control method has been applied on a case study in ATM Congested Network and simulations are conducted, and simulation results will be compared with old method.

*Keywords* — Linear Matrix Inequality (LMI), Switching Control, Multiple bottleneck.

### I. INTRODUCTION

High-speed computer communication networks are generally store-and-forward backbone networks consisting of switching nodes and communication links based on a certain topology. All the links and all the nodes are characterized by their own capacities for packet transmission and packet storing, respectively. A node which reaches its maximum storing capacity due to the saturation of its processors or one or more of its outgoing transmission links is called congested. Some of the packets, arriving at a congested node, cannot be accepted and have to be retransmitted at a later instance. This would lead to a deterioration of the network's throughput and delay performance or even the worst situation—network collapse. Therefore, congestion control is an important problem arising from the networks management [1].

Many algorithms have been proposed for computing explicit rates in single congested node. In general, these algorithms are of two types: the queue length and arrival rate of queue. The stability of the closed-loop system is critical in any congestion control scheme due to the fact that propagation delay encountered in high-speed networks may cause the controllers and the whole network to operate at an unstable point. This yields the notorious oscillation problem that greatly degrades the network performance [2]. But many of these algorithms are not shown to be asymptotically stable in steady state situation,

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and also these algorithms cannot able to use of full capacity of resources in transient states [3].

In this paper a new LMI based switching controller for multiple Bottleneck packet switching Networks has been considered. The main goal is to illustrate the potential impact of the Switching Control methodology [6] on the congestion control problem of the packet switching Networks with dynamically varying parameters and time delays. In addition in order to solve the problems caused by unknown system parameters, robust controllers are used. The congestion dynamics for congested ATM networks is presented in section II. In section III, LMI based switching controller has been discussed. Then, the proposed control method has been applied on a case study and simulations are conducted, compared and discussed in section IV. Finally, in Section V we present our conclusions.

## II. CONGESTION DYNAMICS

Three types of dynamic equations are defined:

1. The equations which are related to the trend of x on which the input rate to the congestion link has affect and its difference with the link transition capacity is c. With only one bottleneck node there is only one x but when there are multiple bottleneck nodes (i.e. when multiple nodes are congested), for each  $x_i$ ,  $i \in \mathbb{N}$  there is a equation for the changes of the queue length in buffer.  $x_i$ ,  $i \in \mathbb{N}$ , denote the number of packets buffered for transmission on link i and  $N = \{1, 2, \dots, n\}$  denote the set of links in network.

2. The calculating allowed sending rate q, and its changes made new equations.

3.All the information about the suggested sending rates are sent to the source by control packets and the applicable traffic rate would be the minimum of the suggested rates and the rate of the source. The three equations of the network for multiple bottleneck nodes are as follows [2]:

$$\begin{aligned} x_{i}^{ab}(n+1) &= x_{i}^{ab}(n) + \min \begin{cases} \psi_{i}^{ab}(n-\tau_{i}^{ab}), & \psi_{i}^{ab}(n-\tau_{i}^{ab}) \\ f_{i}(n) &= f_{i}(n) \end{cases} (X - x_{i}(n) + \psi_{i}(n) \end{cases} - \psi_{i}^{ab}(n) \\ q_{i}(n+1) &= Sat_{q^{0}} \{q_{i}(n) - \sum_{j=0}^{J_{i}} a_{ij}(x(n-j) - x^{0}) - \sum_{k=0}^{K_{i}} b_{ik}q_{i}(n-k) \\ \psi_{e_{ab}}^{ab}(n) &= r_{ab}(n) = \min \begin{cases} q_{m}(n+1 - d_{ma}^{ab}), m \in p(ab); r_{ab}^{0}(n) \end{cases} i \in N \\ (ab) \in C(i) \end{cases} \\ a_{ii} \text{ and } b_{ii} \text{ are the controller gains.} \end{aligned}$$

Where  $x_i^{ab}(n)$  is the queue length for (ab) connection traffic stored in buffer link i in the moment n,  $\psi_i^{ab}(n)$  is the passing traffic of connection (ab) on link i on a time



Figure1. A network with several bottleneck nodes.

interval of [n, n + 1). Queue equations for link i is  $x_i(n+1) = Sat_X \{x_i(n) + f_i(n) - c\}, i \in N$  (2) Where  $x_i(n)$ ,  $f_i(n)$  are the total occupancy of the buffer of link i and the aggregate input flow to link respectively and

#### **III. PROBLEM DEFINITION**

The goal of congestion control in a network is to queue length(x) to be achieved to desire value of  $x_0$ . If a buffer becomes full, it is saturated. Here, the controller objective is to control the number of the packets entering the buffer in order to keep it in a desired value. Suppose that a part of a network is congested. This part may include a number of connected links which are affected by several data streams. Figure 1 shows a part of a network affected by  $r_A$ ,  $r_B$ ,  $r_C$ ,  $r_D$ ,  $r_E$ ,  $r_F$  connection data stream.

#### A. Proposed Method

x is buffer size [4].

In order to design a controller some areas should be defined. These working areas are defined according to the buffer saturation. For each working area a dynamic equations can be defined according to how the buffers are saturated. The best tool for designing the controller that guarantees the robustness of the network is LMI [6].

During the congestion control of the network the areas change and make different dynamics then switching control is used for stabilizing the system [5]. In such type of switching, it is assumed that the system remains in a single working area for a certain period of time.

The Switching Control system based on the LMI theory, used for the design of congestion control scheme, is described In Figure 2. The set of candidate controllers is taken to be LMI should be selected by switching index function that recognized the network area based on network feedback.

# B. Switching

To deal with these types of uncertainties a controller better than linear feedback theory can provide, is obviously required. What is needed is a controller which can change or be changed in response to perceived changes in plant dynamics. If plant changes can be predicted in advance or can be directly measured when they occur, then controller gain scheduling will be often sufficient. But if plant changes cannot be predicted or directly measured, online controller selection or "tuning" must be carried out. The goal of this paper is to describe a simply-structured "highlevel" controller called a "supervisor" which is capable of



Figure.2: congestion control structure

switching into feedback for congestion control of a communication packet switching networks. A sequence of linear positioning controllers from a family of candidate controllers should be identified in order to output of the network approach and track  $x_0$ .

# C. LMI Theory

Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design. See [5] for a good introduction to LMI concepts.

Theory [7]: undetermined closed loop system with the input signal u(t), is totally exponentially stable if symmetric and positive definitive matrix x and a set of matrices  $Q_l$  can be found such that equation (5) holds. The feedback gain needed for stabilizing the closed loop system is expressed in equation (6).

$$u(t) = \sum_{l=1}^{m} \mu_{l} K_{l} x(t) \qquad x(t+1) = \sum_{j=1}^{m} \sum_{l=1}^{m} \mu_{j} \mu_{l} (A_{l} + B_{l} K_{j}) x(t) \quad (3)$$

$$V(x) = x^{T} X^{-1} x \qquad (4)$$

$$\begin{bmatrix} -X & X A_{l}^{T} + Q_{j}^{T} B_{l}^{T} \\ A_{l} X + B_{l} Q_{j} & -X \end{bmatrix} < 0 \qquad (5)$$

$$K_{l} = Q_{l} X^{-1} \qquad (6)$$

## IV. APPLYING THE PROPOSED CONTROL METHOD

The Figure (1) shows a network with several nodes. The traffic applied to each buffer is presented in Table 1 and Table 2. The reason for selecting the second type of the applied traffic is to investigate the efficiency of previously discussed method in the case of changing the working area.

TABLE 1. THE APPLIED TRAFFIC TO B

The applied traffic rate	End time	Start time	Link
0.6	3000	20	1
0.1	2500	50	2
0.1	3000	500	3
0.7	2500	1000	4
0.1	3000	1500	5
0.7	3000	2000	6
TABLE 2. THE APPLIED TRA	FFIC TO C		

The applied traffic rate	End time	Start time	Link
0.9	3000	1500	1
0.6	500	50	2
0.1	1000	500	3
0.9	2500	1500	4
0.2	1200	150	5
0.9	3000	1500	6

According to the system equations of the multiple bottleneck node network presented in [4] and the conditions in the following table also by ignoring the waiting time in the buffer, the initial value of system equations is  $X_0=30$ , C=60,  $\tau_{pr} \ll \tau_s$ , RTD- Sampling time= T, X=100 And 10  $\times \tau_s$  = Propagation delay =  $\tau_p^1$ Now by considering the network in Figure 1 the working area have been defined as below:

- 1) no buffer is saturated
- 2) buffer 2 is saturated
- 3) buffer 3 is saturated
- 4) buffer 1 and buffer 2 are saturated
- 5) buffer 2 and buffer 3 are saturated
- 6) all buffers are saturated

In the following the details of proposed method for some of them have been presented.

# A. If buffer 2 is saturated

Buffer 2 being saturated, a new working area is created in which  $x_2(n) + f_2(n) > c$  and for other buffers,  $i = \{1,3,4\}$   $x_i(n) + f_i(n) \le c$ . The fact that the saturation in one buffer may result in saturation in other buffers is not considered here, because if another buffer becomes saturated due to saturation of buffer2, another working area is created. Here, the control goal is that this working area,  $x_2(n)$ , converges to  $x_0$ . Assuming that buffer 2 is saturated, the dynamic equations is as follows:

 $x_{1,3,4}(n+1) = 0$ 

$$x_2(n+1) = x_2(n) + r_B + r_C + r_A + r_E - C$$

In order to achieve the control objective in this area, a dynamic error is added to the system which expressed by the following equation:

 $e_{2}(n+1) = e_{2}(n) + x_{0} - x_{2}(n)$ 

Since there is no uncertainty in the governing equations of the network, convergence of  $e_2(n)$  to zero is sufficient to achieve the control objective. In order to do, pole placement method and the control rule  $u = k\overline{x}$  should be

applied where the state vector is 
$$\overline{x} = \begin{bmatrix} x_2 & e_2 \end{bmatrix}$$
.  
 $K_{10} = \begin{bmatrix} 0.075 & 0.075 & 0.075 & 0.075 \\ -0.005 & -0.005 & -0.005 \end{bmatrix}^T$ 

## B. If buffer 3 is saturated

Saturation of buffer 3 creates a new working area in which  $x_3(n) + f_3(n) > c$  and for other buffers we have  $i = \{1,2,4\}$   $x_i(n) + f_i(n) \le c$ . According to Figure 1, the

input  $r_D(n)$  and  $r_F(n)$  enter the third buffer immediately and the input  $r_A(n)$  can enter the buffer either immediately or with delay. The bi-aspect behavior of makes us consider it as an uncertainty. The  $r_A(n)$ dynamic equations of the network for this working area are as follows:

$$x_{1,2,4}(n+1) = 0$$

$$x_{3}(n+1) = x_{3}(n) + r_{D}(n) + r_{F}(n) + r_{A}(n) - c + \Delta r_{A}(n)$$

where  $\Delta r_A(n)$  represents the uncertainty in  $r_A(n)$ which may appear as:  $r_{A}(n-4), r_{A}(n-3), r_{A}(n-2), r_{A}(n-1)$ 

Design of a controller for this working area should be based on robust control theories such as LMI. In order to design the dynamic control, the buffer error of  $x_3$  from  $x_1$ 

and the delays related to  $r_A(n)$ , considered as uncertainty, should be added to the dynamic equations of the system. Therefore, the uncertainty in  $\Delta r_A(n)$  is implicitly taken into account in the controller design.

$$x_{3}(n+1) = x_{3}(n) + r_{D}(n) + r_{F}(n) + r_{A}(n) - c + \Delta r_{A}(n)$$
  

$$x_{3}(n-1+1) = x_{3}(n), x_{3}(n-2+1) = x_{3}(n-1)$$
  

$$e_{3}(n+1) = e_{3}(n) + (x_{0} - x_{3}(n))$$
  

$$r_{A}(n-1+1) = r_{A}(n), r_{A}(n-2+1) = r_{A}(n-1)$$
  

$$r_{A}(n-3+1) = r_{A}(n-2), r_{A}(n-4+1) = r_{A}(n-3)$$
  
By using LMI, K is:

-0.0149 -0.0094 -0.0005 -0.0280 -0.4312 -0.3027 -0.0469 -0.0022  $K_{01} = \begin{bmatrix} 0.7730 & 0.0251 & 0.0011 & -0.0927 & 0.2518 & 0.1811 & 0.0291 & 0.0014 \end{bmatrix}$ 0.7730 0.0251 0.0011 -0.0927 0.2518 0.1811 0.0291 0.0014

## C. If both buffer 2 and buffer 3 are saturated

In this case, for buffer 2 we have  $x_2(n) + f_2(n) > c$  and for buffer 3 we have  $x_3(n) + f_3(n) > c$ . Therefore, by adding the new variables, buffer error integral  $x_2$  from  $x_0$  and  $x_3$  from  $x_0$ , the governing equations of the system are as follows:

$$x_{1,4}(n+1) = 0$$
  

$$x_{2}(n+1) = x_{2}(n) + r_{A} + r_{B} + r_{C} + r_{E} - c$$
  

$$x_{3}(n+1) = x_{3}(n) + r_{D} + r_{F} + \delta - c$$

where  $\delta$  is the uncertainty of buffer 3 which can be defined as follows:

$$\delta = \left(\frac{\left(x_{2}^{A} + r_{A}(n-i)\right)}{x_{2}(n) + r_{A} + r_{E} + r_{C} + r_{B}}\right) \qquad i = 0, 1, 2, \dots$$

By adding the new equations we have:

 $e_2(n+1) = e_2(n) + (x_0 - x_2(n)), e_3(n+1) = e_3(n) + (x_0 - x_3(n))$ 

As it can be seen in the above equations, the input to buffer2 and buffer 3 are independent and therefore, distributed controller design methods can be applied for designing the controller. Hence, first for buffer 2 and then for buffer 3 controllers are designed separately. Because in the dynamics of buffer 3, some deterministic terms appear, LMI is used for controller design. Since in this problem the uncertainty has the form of noise and causes instability in the closed loop system, for  $x_2$  it is bounded. Therefore, the problem is to design a decentralized controller for which the state feedback gain is as follows:

$$K_{11} = \begin{bmatrix} 0.075 & 0 & -0.005 & 0 \\ 0.075 & 0 & -0.005 & 0 \\ 0.075 & 0 & -0.005 & 0 \\ 0 & 0.15 & 0 & -0.01 \\ 0.075 & 0 & -0.005 & 0 \\ 0 & 0.15 & 0 & -0.01 \end{bmatrix}$$

# V. SIMULATION

Considering above, for the presented example there are four working areas: The working area in which only the second buffer becomes saturated, the working area in which only the third buffer becomes saturated, the working area in which the second and third buffer become saturated and the working area in which none of buffers become saturated. For each of these working areas the appropriate controller is designed based on the robust control theory and by applying the silence time switching logic the appropriate controller is selected and used.

By applying these controllers and using the switching controller, the following results for the applied traffic are presented in Table 1 and Table 2. Figure 3 - 6 depict the results of comparison between the obtained results and the response of the closed loop system controlled by a single controller. The comparison criteria is the response of the closed loop system according to the output performance and the control signal.

Figure 3 depicts the response of the closed loop system when the traffic is applied according to Table 1. By comparing between the closed loop system response, it is obvious that the performance of the closed loop system is improved by using the proposed method and the controller signal is also enhanced (Figure 4).

Figure 5 shoes the closed loop system response when the traffic is applied according to Table 2. By studying the response of the closed loop system it is obvious that the proposed method not only improves the performance of the closed loop system, but also the old method shows a weak performance and therefore by using the proposed controller the control signal is enhanced to a large extent.



Figure 3. the output of the third buffer by using previous and the proposed methods



Figure 4. The resulted control signal of the previous and the proposed methods



Figure 5. The output of the third buffer using previous and proposed methods



Figure 6. Resulted control signal by using previous and proposed method

## VI. CONCLUSION

In this paper a new LMI based switching controller for multiple Bottleneck packet switching Networks was presented. The main goal was to illustrate the impact of the Switching Control methodology on the congestion control problem of the packet switching Networks with dynamically varying parameters such as link capacity (c) and time delays. Depends on network condition different working area defined and for each of these working areas the appropriate controller was designed based on the robust control theory and by applying the silence time switching logic. Simulation result depicts that by considering the output response of the closed loop system, it is obvious that the proposed design method results in an enhancement in the performance of the closed loop system and by using the proposed controller the control signal is enhanced.

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