Power Allocation in OFDM-Based Cognitive Radio Systems Considering PAPR Constraint

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Abstract— **The problem which is studied in this article is maximizing the capacity between two paired cognitive radios, considering peak to average power ratio (PAPR). The powers of the subcarriers are the unknown parameters of the optimization problem. To exert the PAPR limit, we have used the Complementary Cumulative Distribution Function (CCDF) of PAPR which is a function of subcarrier's power. Using the function, we need PAPR to be greater than γ with the probability less than a threshold β. The achieved optimization problem is solved using numerical methods.**

Keywords- **cognitive radio, optimization, capacity, OFDM, PAPR;**

I. INTRODUCTION

The ever increasing demand to achieve higher data rates in recent years, have redouned to frequency scarcity. The most important element caused this problem is assigning a specific spectrum band to Primary Users (PU) which they do not use it all time. To solve this problem, J. Mitola III in [1] offered Software Radios (SR) as a kernel which is implemented over Cognitive Radios (CR or Secondary User (SU)). CRs intelligently sense the environment, and send data according to the collected information.

One of the most important cases in CR's implementation is how to send data over spectrum holes sensed by the user. In several references like [2] and [3], OFDM because of its flexibility, having many streaky variables to be altered adaptively and also being less complex comparing to other methods, is offered as the best choice to send data over the communication channel.

One of the major problems in OFDM systems is the problem of PAPR. Superposition of a large number of subcarriers with different phases causes large peaks in output signal. If this vast variations of signal's amplitude is not in the range of linear operation of circuit elements like A/D or D/A, the output signal would have unacceptable distortions. Therefore we need a method to decrease and control the amount of PAPR. Many researchers have developed estimation of PAPR's probabilistic features. Articles published before like [4], have calculated CCDF of PAPR just for equal subcarrier power. In [4] CCDF is estimated precisely for unequal subcarrier's power. Using the resulted function we can exert the PAPR limit as one of the limitations of the power allocation problem to maximize the capacity of the channel.

The remaining of this paper contains four sections. Section II is dedicated to the system model as a network of PUs and CRs which affect each other. In section III the formulation of PAPR as a constraint of the optimization problem is derived. In section IV the simulation results would be presented, and finally section V gives concluding remarks.

II. SYSTEM MODEL

Here we have two simplification assumptions. First only two paired CR users assumed to be in the network and other users are PUs. Second, bit Loading algorithm has not been applied in the transmitter. Single CR pair and PUs are depicted in Fig.1.
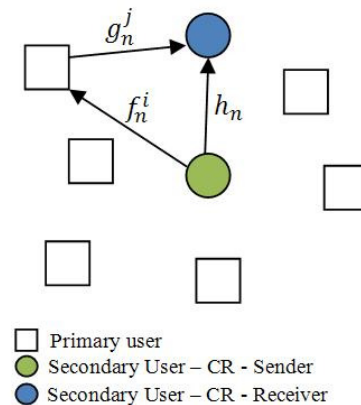

Figure 1: Network consisted of 2 paired CRs and many PUs

The coefficients h_n, g_n^j, and f_n^i pertain to channel coefficients between the two SUs, the j-th PU sender and the SU receiver and finally the SU sender and i-th PU receiver, respectively, all for the n-th subchannel. The problem is to

maximize the channel capacity between the two SUs considering the related constraints. Assuming that the channel is Gaussian, the channel capacity is presented in (1) due to [5].

$$C = \sum_n \frac{1}{2} \log\left(1 + \frac{p_n h_n}{I_n}\right) \tag{1}$$

$$I_n = N_0 + \sum_{i=1}^{K_{PU}} q_n^i g_i^2 \tag{2}$$

Which in (2), I_n is the total noise power equals with the sum of Gaussian noise power, N_0, and the effect of all PUs at the SU receiver, on the n-th subcarrier. p_n is the power of the SU on the n-th subcarrier, q_n^i is the power of the i-th PU on the n-th subcarrier, and K_{PU} is the total number of the subcarriers.

The resulting capacity function would be the object function of the optimization problem and is moderated by the PAPR constraint which is calculated in the next section.

III. PROBLEM FORMULATION

According to the results of [4], PAPR CCDF of OFDM signal for the case which power of subcarriers is not equal would be

$$Pr[PAPR > \gamma] \approx 1 - exp\left\{-2e^{-\gamma}\sqrt{\frac{\pi\gamma}{NP_{av}}\sum_{k=-K}^{K} k^2 p_k}\right\} \tag{3}$$

Where

$$P_{av} = \frac{1}{2K}\sum_{k=-K}^{K} p_k \tag{4}$$

$$\begin{cases} K = \dfrac{N_{active}}{2} & if\ N_{active}\ is\ even \\ K = \dfrac{N_{active} - 1}{2} & if\ N_{active}\ is\ odd \end{cases} \tag{5}$$

In the above equations p_k is the power of the k-th subcarrier. N is the total number of subcarriers, P_{av} is the average power of the transmitter, γ is the power threshold, and N_{active} is the number of active subcarriers. To use PAPR in the optimization problem we have the following relation:

$$Pr[PAPR < \gamma] > \beta \tag{6}$$

In other words, we need the PAPR to be greater than γ with the probability less than β. According to (3), the PAPR condition of (6) is simplified as:

$$\sum_{k=-K}^{K}\left(k^2 - \frac{(Ne^{2\gamma}\ln^2\beta)}{4\pi\gamma}\right)p_k < 0 \tag{7}$$

To define the optimization problem, as discussed in section II, the capacity function is considered as the object function. The constraints would be the resulting PAPR condition in (7), noise temperature threshold at the receiver, and total power

constraint. We define $\mu_n = \frac{p_n}{I_n}$ as the SNR of the n-th subchannel at the transmitter. It is obvious what varies μ_n is the noise power I_n, because the h_n value is a constant and p_n is specified in the problem. Thus the resulting optimization problem would be:

$$\max C = \sum_n \frac{1}{2}\log(1 + \mu_n h_n) \tag{8}$$

$$subject\ to: \begin{cases} \displaystyle\sum_{n=-K}^{K} (n^2 - B)p_n \le 0 & (9) \\ 0 \le p_n \le p_n^0, n = -K, \dots, K & (10) \\ \displaystyle\sum_{n=-K}^{K} p_n \le P_T & (11) \end{cases}$$

Where:

$$B \triangleq \frac{Ne^{2\gamma}\ln^2\beta}{4\pi\gamma} \tag{12}$$

$$p_n^0 \triangleq \min_i\left\{\frac{T_0 - N_0}{f_n^i}\right\} \tag{13}$$

Which p_n^0 is the acceptable power over the n-th subcarrier according to its effect as noise on the SU receiver. T_0 is the noise temperature threshold, and N is the total number of subcarriers.

One important point about (9) is the effect of B on the productivity of the constraint. The minimum value of B that make (9) feasible for any value of p_ns can be calculated as below:

$$\sum_{n=-K}^{K} (n^2 - B)p_n \le 0 \tag{14}$$

$$\frac{1}{P_T}\sum_{n=-K}^{K} n^2 p_n \le B \tag{15}$$

$$B_{min} = \max\left(\frac{1}{P_T}\sum_{n=-K}^{K} n^2 p_n\right) = 4K^2 \tag{16}$$

Therefore if the value of B becomes greater than B_{min} then (9) is no longer affect the results.

The convexity of the problem is obvious due to linear constraints and convexity of the capacity function. In the next section the numerical results of the problem will be introduced.

IV. SIMULATION RESULTS

In this section the simulation results of the proposed problem is introduced. The presented problem in (8-11) is solved numerically using MATLAB optimization toolbox. The total number of subcarriers $N = 64$ is considered and $P_T = 64$. The Channel State Information (CSI) for 64 subchannels

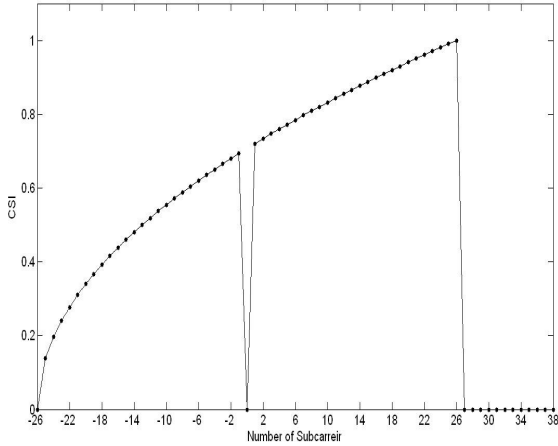is depicted in Fig.2 according to the channel used in [4] for IEEE 802.11a standard.

Figure 2: CSI for the simulated program

Using these CSI coefficients for each subcarrier, the value of h_n for different ns is specified and $K = 26$ is obtained. The simulation is done for various SNRs and different values of γ for $\beta = 0.95$. In Tab.1 separate simulation adjustments and its corresponding figure number is presented.

Table 1 : parameter adjustments for the simulation

γ	β	B	SNR	*Fig. No.*
4		9.98559	-10	Fig 3.a
4.5		24.1284	0	Fig 3.b
5	0.95	59.0291	10	Fig 3.c
5.5		145.8706	20	Fig 3.d
6		363.4744		
N/4 = 16	0.95	6.6129e+10	-10:10:20	Fig.4
Note: N=64, K=26, CSI according to Fig.2.				

Using these values the simulation results are depicted in Fig.3(a,b,c,d) and Fig.4.

In Fig.3 the effect of γ variations which directly affects B is considered for different SNRs. In this figure the variables are chosen such that the value of B varies in the range that PAPR constraint in (9) strongly affects the result of the problem solution.
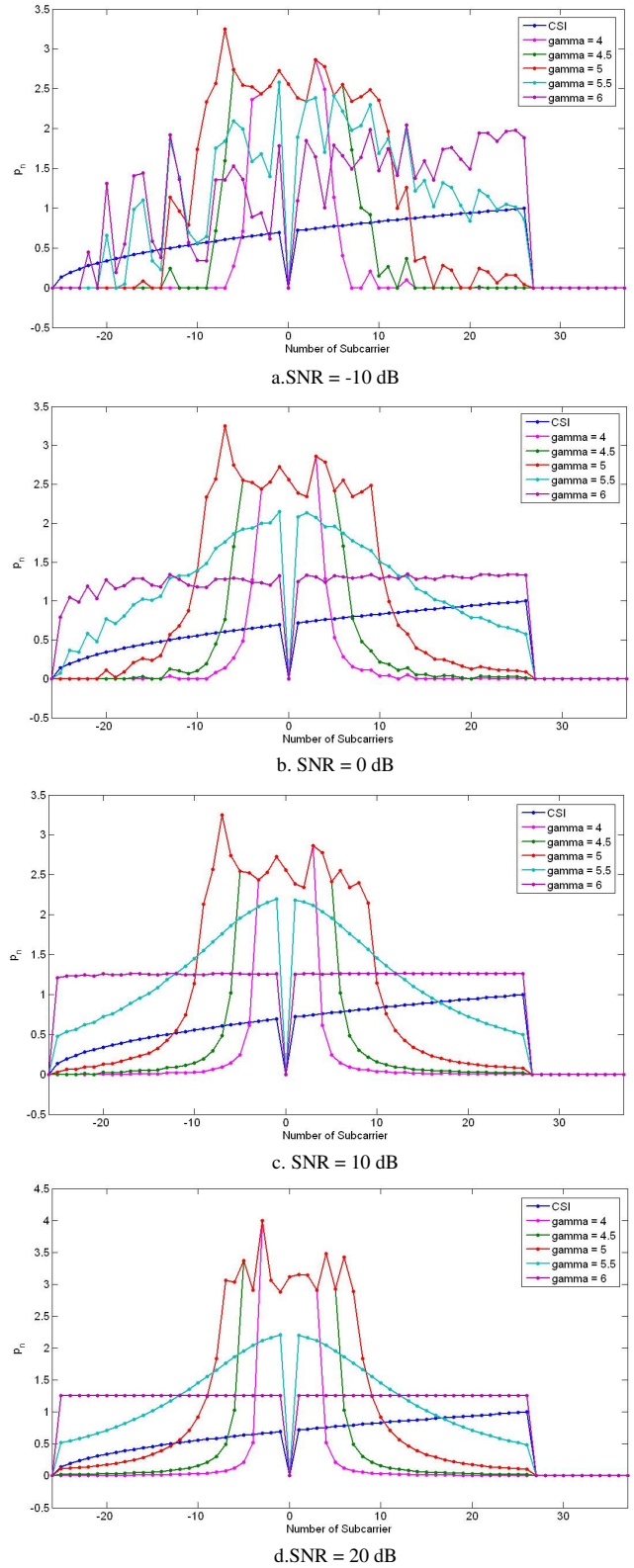

a.SNR = -10 dB

b. SNR = 0 dB

c. SNR = 10 dB

d.SNR = 20 dB

Figure 3 : The effect of different γ values on power allocation of OFDM subcarriers considering PAPR constraint for different SNRs

As could be seen in Fig.3, for small values of γ the effective subcarriers are limited to the near zero band and as γ grows (so B) the band extends and approaches to a state that no PAPR constraint is considered. As SNR grows from -10dB to 20dB the local fluctuations of the power values are decreased and the curves become smoother.
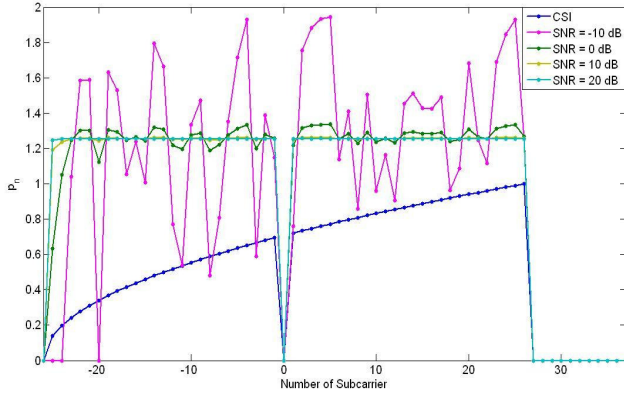

Figure 4: The effect of SNR on power allocation of OFDM subcarriers while the PAPR constraint is not affect the optimization problem

Inasmuch as in Fig.4 the value of $B \gg B_{min}$ the PAPR constraint is no longer affect the problem and the power allocation will simply obey the waterfilling algorithm. Like before as SNR grows the power values become smoother.

Solving the optimization problem using numerical methods, the 3D diagram of optimized capacity as a function of SNR and the probability of β for different γs is obtained. The related diagram is depicted in Fig.5.
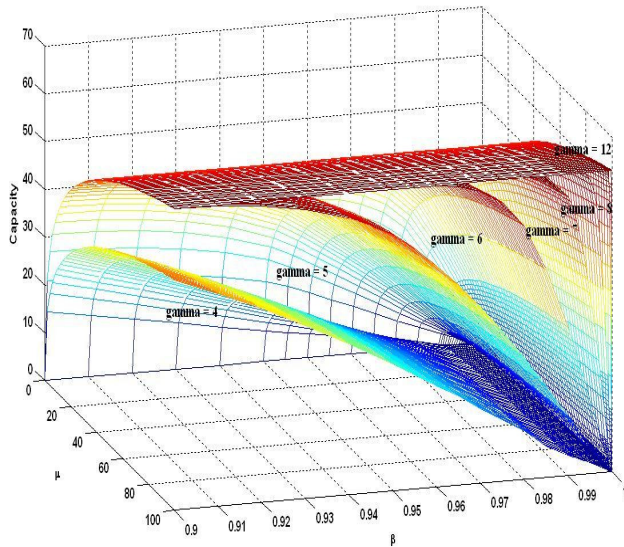

Figure 5: Optimum capacity of the channel in relation to μ, and β for different γ

According to Fig.5 as γ and the probability value of β increases the capacity surfaces approach to a surface which is

defined as an optimum surface resulted from the optimization problem (8-11), excluding the PAPR condition. The main reason is the dependency of B to the values of γ and β according to (12). In Fig.6, B is depicted as a function of γ and β.
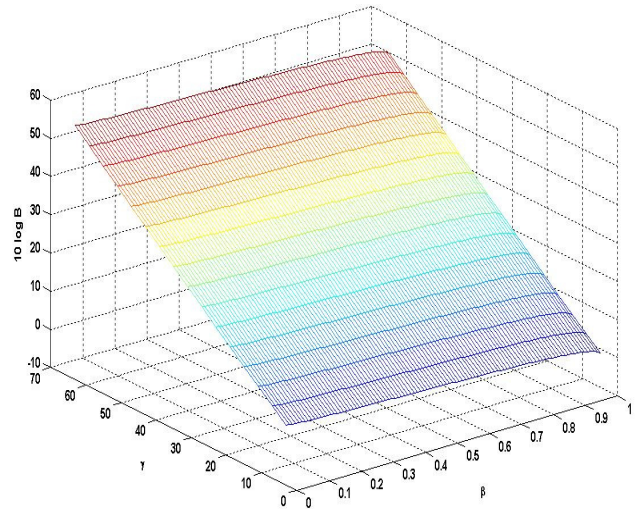

Figure 6: Relation of B with γ and β

As calculated in (16) for B values greater than B_{min} the PAPR constraint in (9) does not affect the solution anymore. According to Fig.6 whereas B is a function of γ and β, we can always choose a proper (γ,β) pair which produce a B greater than B_{min}. Therefore the problem will simplified as an optimization problem with two constraints as (10) and (11).

V. CONCLUSIONS

In this paper a new power allocation method for OFDM based cognitive radios which considers PAPR as a constraint was presented. The optimization problem was numerically solved and the effect of PAPR constraint on capacity was studied. For $B < B_{min}$ the PAPR constraint affects the results such that for small values of B the near zero subcarriers attain more power. Also there was a direct relation between the bandwith of the effective subcarriers and amount of B. For $B \geq B_{min}$ which could be achieved for some values of β and γ the problem becomes independent of the PAPR constraint and it would be simplified.

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